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On the Measurement of Inequality under Uncertainty*

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To take into account both ex ante and ex post inequality considerations, one has to deal with inequality and uncertainty simultaneously. Under certainty, much of the literature has focused on "comonotonically linear" indices: functionals that are linear on cones of income profiles that agree on the social ranking of the individuals. This family generalizes both the Gini index and the egalitarian index (minimal income). However, it does not include functionals such as the average of expected-Gini and Gini-of-expectation. In contrast, the family of min-of-means functionals is rich enough for this purpose. *Journal of Economic Literature* Classification Numbers: D31, D63, D81.

1. MOTIVATION

The bulk of the literature on inequality measurement assumes that the income profiles do not involve uncertainty. It is natural to suppose that if uncertainty (or risk) is present, one may use the theory of decision under uncertainty to reduce the inequality problem to the case of certainty, say, by replacing each individual's income distribution by its expected value or expected utility. Alternatively, it would appear that one may use the theory of inequality measurement to reduce the problem to a single decision-maker's choice under uncertainty, say, to the choice among distributions

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over inequality indices. We claim, however, that neither of these reductions would result in a satisfactory approach to the measurement of inequality under uncertainty. Rather, inequality and uncertainty need to be analyzed in tandem. The following example illustrates.

Consider a society consisting of two individuals, *a* and *b*. They are facing two possible states of the world, *s* and *t*. A social policy determines the income of each individual at each state of the world. We further assume that both individuals, hence also a "social planner," have identical beliefs represented by a probability over the states. Say, the two states are equally likely. Consider the following possible choices (or "social policies").

f_1	a	b
s	0	0
t	1	1

f_2	a	b
s	1	1
t	0	0

g_1	a	b
s	0	1
t	1	0

g_2	a	b
s	1	0
t	0	1

h_1	a	b
s	0	1
t	0	1

h_2	a	b
s	1	0
t	1	0

We argue that a reasonable social ordering $\geqslant \cdot$ would rank these choices from top to bottom: $f_1 \approx \cdot f_2 > \cdot g_1 \approx \cdot g_2 > \cdot h_1 \approx \cdot h_2$. Indeed, symmetry between the states and anonymity of individuals imply the equivalence relations. The *f* choices are preferred to the *g* choices due to *ex-post inequality*: in all of these (*f* and *g*) alternatives, the expected income of each individual is 0.5, and thus there is no inequality ex-ante. But according to *f*, both individuals will have the same income at each state of the world, while under *g* the resulting income profile will have a rich individual and a poor individual.[1]

One may argue that the *f* alternatives are riskier than the *g* ones from a social standpoint, since under *f* there is a state of the world in which no individual has any income, whereas the *g* alternatives allow additional transfers in each state of the world, after which both individuals would have a positive income. However, we consider a social planner's preferences

[1] Myerson [10] also draws the distinction between ex-ante and ex-post inequality, and argues that a social planner's preference for ex-post inequality might lead to a choice which is not ex-ante Pareto optimal.

over *final* allocations. These preferences are the basis on which potential transfers will be made.

The comparison between g and h hinges on *ex-ante inequality*: ex-post, both choices have the same level of inequality at each state of the world. However, the g alternatives promise each individual the same expected income, while the h choices pre-determine which individual will be the rich and which will be the poor one. Thus g is "more ex-ante egalitarian" than h. Matrices g_2 and h_2 are identical to those of [4][2] .

We observe that one cannot capture these preferences if one reduces uncertainty to, say, expected utility and measures the inequality of the latter, or vice versa. For instance, suppose that the Gini index is the accepted measure of inequality. In the case of two individuals, and in the absence of uncertainty, the Gini welfare function can be written as $G(y_1, y_2) = [3\tilde{y}_1 + \tilde{y}_2]/4$ where $(\tilde{y}_1, \tilde{y}_2)$ is a permutation of (y_1, y_2) such that $\tilde{y}_1 \leqslant \tilde{y}_2$. Ranking alternatives by the Gini welfare function of the expected incomes will distinguish between g and h, but not between f and g. On the other hand, selecting the expected Gini index as a choice criterion will serve to distinguish between f and g, but not between g and h. By contrast, a (weighted) average of the expected Gini index and the Gini of the expected income would rank f above g and g above h. We are therefore interested in measures of social welfare under uncertainty that take into account both ex-ante and ex-post inequality, and, in particular, that include the above-mentioned functionals.

Furthermore, our goal is to characterize a class of measures that is a natural generalization of those commonly used for the measurement of social welfare under certainty. That is, we seek a set of principles that will be equally plausible in the contexts of certainty and of uncertainty, that is satisfied by known measures under certainty, and that, under uncertainty, will reflect both ex-ante and ex-post inequality considerations.

2. INEQUALITY AND UNCERTAINTY

The relationship between the measurement of inequality and the evaluation of decisions under uncertainty has long been recognized. Harsanyi's utilitarian solution [8] corresponds to maximization of expected utility, while Rawls' egalitarian solution [12] is equivalent to maximization of the minimal payoff under uncertainty.[3] Further, the previous decade has

[2] One can justify the preference of g over h on grounds of "procedural justice" [9]: it is more just that Nature (i.e., a lottery) would choose whether a and b will be the rich individual, rather than that the choice be made by a person (or persons) acting on behalf of "society."

[3] Indeed, Rawls' conceptual derivation of his criterion resorts to the "veil of ignorance," that is, to the reduction of social choice problems to decision under uncertainty. (See also [7].)

witnessed several derivations of a class of functionals that generalizes both expected utility (in the context of choice under uncertainty) and the Gini index of inequality (in the context of inequality measurement). The reader is referred to [2, 11, 13–16]. Chew also pointed out the relationship between the "rank-dependent probabilities" approach to uncertainty and the generalization of the Gini index.

The "rank-dependent probabilities" approach suggests that the probability weight assigned to a state of the world in the evaluation of an uncertain act f depends not only on the state, but also on its relative ranking according to f. Yet, if we restrict our attention to acts that are "comonotonic," i.e., that agree on the payoff-ranking of the states, probabilities play their standard role as in expected utility calculations. We therefore refer to these functionals as "comonotonically linear."

For simplicity, consider the symmetric case. A symmetric comonotonically linear functional would be characterized by a probability vector $(p_i)_{i=1}^n$, where there are n states of the world. Given an uncertain act f, that guarantees a payoff of f_j at state of the world j, let $f^{(i)}$ be the ith lowest payoff in f. Then, f is evaluated by a weighted sum

$$I(f) = \sum_i p_i f^{(i)}.$$

In the context of inequality measurement, [2, 14, 16] considered the same type of evaluation functionals, where an element j is interpreted as an individual in a society, rather than as a state. In this context, it is natural to impose the symmetry (or "anonymity") condition, which implies that an individual's weight in the comonotonically linear aggregation above depends *only* on her social ranking. A common assumption is that $p_1 \geqslant p_2 \geqslant \cdots \geqslant p_n$. For such a vector p, the functional above represents social preferences according to which a transfer of income from a richer to a poorer individual, that preserves the social ranking, can only increase social welfare.

Special cases of these functionals are the following: (i) if $p_i = 1/n$, we get the average income function; (ii) if $p_{i-1} - p_i = p_i - p_{i+1} > 0$ (for $1 < i < n$), the resulting functional agrees with the Gini index on subspaces of income profiles defined by a certain level of total income (see [1]); (iii) if $p_1 = 1$ (and $p_i = 0$ for $i > 1$), the functional reduces to the minimal income level. While axiomatizations of these special cases do exist in the literature, we prefer to keep the discussion here on the more general level, dealing with all functionals defined by $p_1 \geqslant p_2 \geqslant \cdots \geqslant p_n$ as above, and focusing on these cases as examples and reference points.

The strong connection between inequality measurement and decision under uncertainty, and, furthermore, the fact that comonotonically linear functionals were independently developed in both fields, may lead one to believe that the problem of inequality measurement under uncertainty is (mathematically) a special case of the known problems of the measurement of inequality—or of uncertainty—aversion. But this is not the case. The rank-dependent approach of Weymark, Quiggin, Yaari, and Chew cannot satisfactorily deal with the preference patterns described in Section 1. Specifically, in each of the six alternatives in the f, g, and h matrices there are two 1's and two 0's. If we follow the rank-dependent approach, applied to the state–individual matrix, and impose symmetry between the states and between the individuals, we will have to conclude that all six alternatives are equivalent. Indeed, the pattern of preferences between the f's and the g's, as well as that between the g's and the h's, is mathematically equivalent to Ellsberg's paradox (see [5]), and the rank-dependent approach is not general enough to deal with this paradox.

We suggest to measure inequality under uncertainty by the class of min-of-means functionals. A min-of-means functional is representable by a set of probability vectors (or matrices in the case of uncertainty) in the following sense: for every income profile, the functional assigns the minimal expected income, where the expectation is taken separately with respect to each of the probability vectors (matrices). Observe that a comonotonically linear functional defined by $(p_i)_{i=1}^n$ with $p_1 \geqslant p_2 \cdots \geqslant p_n$ can be viewed as a min-of-means functional, for the set of measures which is the convex hull of all permutations of $(p_i)_{i=1}^n$.

In Section 3 we define this class axiomatically, and contend that the axioms are acceptable in the presence of uncertainty no less than under certainty. After quoting a representation theorem, we prove (in Section 4) that this class is closed under iterative application, as well as under averaging. It follows that this class includes linear combinations of, say, the expected Gini index and the Gini index of expected income. We devote Section 5 to briefly comment on the related class of Choquet integrals. Finally, an appendix contains an explicit calculation of the probability matrices for some examples.

3. MIN-OF-MEANS FUNCTIONALS

We start with some general-purpose notation. For a finite set A, let $F = F_A$ be the set of real-valued functions on A, and let $P = P_A$ be the space of probability vectors on A. For $f \in F$ and $p \in P$ we use f_i and p_i with the obvious meaning for $i \in A$. Elements of F and of P will be identified with real-valued vectors or matrices, as the case may be. We also use $p \cdot f$ to denote the inner product $\sum_{i \in A} p_i f_i$.

Specifically, let S be a finite set of states of the world, and let K be a finite set of individuals. (Both S and K are assumed non-empty.) Let $A = S \times K$, so that $F = F_A$ denotes the space of income profiles under uncertainty for the society K where uncertainty is represented by S.

Let $\geqslant\cdot$ denote the binary relation on F, reflecting the preference order of "society" or of a "social planner." Consider the following axioms on $\geqslant\cdot$. (Here and in the sequel, $\approx\cdot$ and $>\cdot$ stand for the symmetric and antisymmetric parts of $\geqslant\cdot$, respectively.)

A1. *Weak order*: for all $f, g, h \in F$: (i) $f \geqslant\cdot g$ or $g \geqslant\cdot f$; (ii) $f \geqslant\cdot g$ and $g \geqslant\cdot h$ imply $f \geqslant\cdot h$;

A2. *Continuity*: $f^k \to f$ and $f^k \geqslant\cdot (\cdot\leqslant)\, g$ imply $f \geqslant\cdot (\cdot\leqslant)\, g$;

A3. *Monotonicity*: $f_{si} \geqslant (>)\, g_{si}$ for all $(s, i) \in A$ implies $f \geqslant\cdot (>\cdot)\, g$;

A4. *Homogeneity*: $f \geqslant\cdot g$ and $\lambda > 0$ imply $\lambda f \geqslant\cdot \lambda g$;

A5. *Shift covariance*: $f \geqslant\cdot g$ implies $f + c \geqslant\cdot g + c$ for any constant function $c \in A$;

A6. *Concavity*: $f \approx\cdot g$ and $\alpha \in (0, 1)$ implies $\alpha f + (1-\alpha)\, g \geqslant\cdot f$.

We do not insist that any of these axioms, let alone all of them taken together, are indisputable. However, under certainty (where F is the set of income vectors), they are satisfied by utilitarian preferences, by egalitarian (maxmin) preferences, as well as by any preferences that correspond to a comonotonically linear functional with $p_1 \geqslant p_2 \cdots \geqslant p_n$. Moreover, axioms 1–6 seem to be as reasonable in the case of uncertainty as they are in the case of certainty.

In [6] it is shown that a preference order satisfies A1–A6 iff it can be numerically represented by a functional $I\colon F \to \Re$ defined by a compact and convex set of measures $C \subseteq P$ as

$$I(f) = \mathrm{Min}_{p \in C}\, p \cdot f \qquad \text{for all} \quad f \in F.$$

Moreover, in this case the set C is the unique compact and convex set of measures satisfying the above equality for all $f \in F$. We refer to such a functional I as a "min-of-means" functional: for every function f, its value is the minimum over a set of values, each of which is a weighted-average of the values of f.

4. ITERATION AND AVERAGING

We now prove that the class of min-of-means functionals is closed under two operations: pointwise averaging over a given space, and iterated application over two spaces. (This, of course, will prove closure under any finite number of iterations.)

Let there be given two sets A_1 and A_2, to be interpreted as the sets of states and of individuals, respectively. Consider the product space $A = A_1 \times A_2$. Given two min-of-means functionals I_1 and I_2 (on $F_1 = F_{A_1}$ and on $F_2 = F_{A_2}$, respectively), we wish to show that applying one of them to the results of the other generates a min-of-means functional on $F = F_A$. We first define iterative application formally.

Notation. For a matrix $f \in F$, define $\tilde{f}_1 \in F_1$ to be the vector of I_2-values of the rows of f; that is, for $i \in A_1$, $(\tilde{f}_1)_i = I_2(f_{i\bullet})$. Then

$$(I_1 * I_2)(f) = I_1(\tilde{f}_1).$$

Should $I_1 * I_2$ be a min-of-means functional (on F), it would have a set of probability matrices (on A) corresponding to it. In order to specify this set, we need the following notation. First, let $P_r = P_{A_r}$ for $r = 1, 2$. We now define a "product" operation between a probability vector on A_1, and a matrix on A, associating a probability vector on A_2 for each element of A_1:

Notation. Let $m = (m_{ij})_{ij}$ be a stochastic matrix, such that $m_{i\bullet} \in P_2$ for every $i \in A_1$. Then, for $p_1 \in P_1$ let $p_1 * m$ be the probability matrix on A defined by

$$(p_1 * m)_{ij} = (p_1)_i \, m_{ij}.$$

Notation. Let $C_r \subseteq P_r$ be given (for $r = 1, 2$). Let $C_1 * C_2 \subseteq P = P_A$ be defined by

$$C_1 * C_2 = \{ p_1 * m \mid p_1 \in C_1, \forall i, m_{i\bullet} \in C_2 \}.$$

That is, $C_1 * C_2$ denotes the set of all probability matrices for which every conditional probability on A_2, given a row in A_1, is in C_2, and whose marginal on A_1 is in C_1.

THEOREM. *Let there be given A_1 and A_2 as above, and let I_1 and I_2 be min-of-means functionals on them, respectively. Let $C_r \subseteq P_r$ be the set of measures corresponding to I_r, $r = 1, 2$, by the result quoted above. Then $I_1 * I_2$ is a min-of-means functional on A. Furthermore, the set $C \subseteq P$ corresponding to $I_1 * I_2$ is $C \equiv C_1 * C_2$.*

Proof. Observe that the set C is compact. We note that it is also convex. Indeed, assume that $p_1 * m$, $p'_1 * m' \in C$ and let $\alpha \in [0, 1]$. Define

$$\bar{p}_1 = \alpha p_1 + (1 - \alpha)\, p'_1 \in C_1$$

and

$$\bar{m}_{i\bullet} = \frac{\alpha(p_1)_i\, m_{i\bullet} + (1-\alpha)(p_1')_i\, m_{i\bullet}'}{\alpha(p_1)_i + (1-\alpha)(p_1')_i} \in C_2$$

for $i \in A_1$ whenever the denominator does not vanish. (The definition of $\bar{m}$ is immaterial when it does.) It is easily verified that

$$\bar{p}_1 * \bar{m} = \alpha(p_1 * m) + (1-\alpha)(p_1' * m').$$

We now turn to show that C is the set of measures corresponding to $I_1 * I_2$. We need to show that for every $f \in F$,

$$(I_1 * I_1)(f) = \mathrm{Min}_{p \in C}\, p \cdot f.$$

Let $f \in F$ be given. We first show that $(I_1 * I_2)(f) \geqslant \mathrm{Min}_{p \in C}\, p \cdot f$. Let $m_{i\bullet} \in C_2$ be a minimizer of $\sum_j m_{ij} f_{ij} \equiv \hat{f}_i$. Let $p_1 \in C_1$ be a minimizer of $p_1 \cdot \hat{f}$ (where $\hat{f}$ is defined in the obvious way). Note that $p_1 \cdot \hat{f} = (I_1 * I_2)(f)$. Since $p_1 * m$ appears in C, the inequality follows.

Next we show that $(I_1 * I_2)(f) \leqslant \mathrm{Min}_{p \in C}\, p \cdot f$. Assume that the minimum on the right-hand side is attained by the measure $p_1 * m$. We claim that, unless $(p_1)_i = 0$, $m_{i\bullet}$ is a minimizer, over all $p_2 \in C_2$, of $\sum_j (p_2)_j f_{ij}$. Indeed, were one to minimize $p \cdot f$ by choosing a measure $p_1 * m$, one could choose $m_{i\bullet}$ independently at different states i; hence, for any choice of p_1, the minimal product will be obtained for $m_{i\bullet}$ that is a pointwise minimizer. Without loss of generality we may therefore assume that $m_{i\bullet}$ is a minimizer of $\sum_j m_{ij} f_{ij} \equiv \hat{f}_i$. Hence p_1 has to be a minimizer of $p_1 \cdot \hat{f}$, and the equality has been established. Finally, since $I_1 * I_2$ is representable as $\mathrm{Min}_{p \in C}\, p \cdot f$, it is a min-of-means functional. ▮

Under the above conditions, both $I_1 * I_2$ and $I_2 * I_1$ are min-of-means functionals. However, in general they are not equal, as the examples in Section 1 show. Specifically, let $A_1 = \{s, t\}$, $A_2 = \{a, b\}$, and define I_1 by $C_1 = \{(1/2, 1/2)\}$ and I_2 by $C_2 = \{(p, 1-p) \mid 0 \leqslant p \leqslant 1\}$. That is, I_1 is the expectation with respect to a uniform prior, and I_2 is the minimum operator. Consider the matrix g_1 defined in Section 1, and observe that $(I_1 * I_2)(g_1) = 0$ while $(I_2 * I_1)(g_1) = 1/2$.

The theorem above states that if a certain inequality index, such as the Gini index or the minimal income index, is representable as a min-of-means functional, so will be that index applied to expected income, and so will be the expected value of this index. However, if we consider a sum (or an average) of these two, we need the following result to guarantee that the resulting functional is also a min-of-means functional.

PROPOSITION. *Let there be given two min-of-means functionals I^1 and I^2 on $F = F_A$. Let $\alpha \in [0, 1]$. Then $I = \alpha I^1 + (1 - \alpha) I^2$ is a min-of-means functional. Furthermore, if C^1 and C^2 are the sets of measures corresponding to I^1 and I^2, respectively, then the set C corresponding to I is given by*

$$C = \{\alpha p^1 + (1 - \alpha)\, p^2 \mid p^1 \in C^1,\, p^2 \in C^2\}.$$

5. A COMMENT ON CHOQUET INTEGRATION

Schmeidler [13] suggested to use Choquet integration [3] with respect to non-additive measures for the representation of preferences under uncertainty. For the sake of the present discussion, the reader may think of a Choquet integral as a continuous functional, which is linear over each cone of comonotonic income profiles. Specifically, assume that f and g are two matrices, with rows corresponding to states, and columns—to individuals. Recall that the two are comonotonic if there is an ordering of the state-individual pairs that both f and g agree with, i.e., if there are no two pairs (s, a) and (t, b) such that $f_{sa} > f_{tb}$ while $g_{sa} < g_{tb}$. In this case, the Choquet integral I with respect to any non-additive measure has to satisfy

$$I(f + g) = I(f) + I(g).$$

Under certainty, the assumption of symmetry (between individuals) reduces the non-additive integration approach to the rank-dependent one. This is not the case, however, when the relevant space is a product of states and individuals. In a two-dimensional space, symmetry between rows and between columns does not imply that every subset of the matrix is equivalent to any other subset with identical cardinality. Thus, the non-additive approach may explain preferences as in Ellsberg's paradox, and can, correspondingly, account for preferences as in Section 1, without violating the symmetry assumptions. This would give one a reason to hope that this approach is the appropriate generalization of comonotonically linear functionals in the case of certainty. Yet, we find that this linearity property, even when restricted to comonotonic profiles, is hardly plausible in our context. Consider the following four alternatives.

f_3	a	b
s	1	0
t	0	0

f_4	a	b
s	0	1
t	0	0

g_3	a	b
s	2	1
t	1	0

g_4	a	b
s	1	2
t	1	0

By symmetry between the two individuals, the first two alternatives are equivalent. Assuming a Choquet-integral representation, this would imply that the last two alternatives are also equivalent. To see this, note that $g_3 = f_3 + h$ and $g_4 = f_4 + h$ for h given by

h	a	b
s	1	1
t	1	0

Since f_3 and h are comonotonic, $I(g_3) = I(f_3) + I(h)$. Similarly, $I(g_4) = I(f_4) + I(h)$. From $I(f_3) = I(f_4)$ one therefore obtains $I(g_3) = I(g_4)$.

However, g_3 and g_4 are equivalent only as far as ex-post inequality considerations are concerned. Ex-ante, g_3 makes one individual always better off than the other, while g_4 guarantees the two individuals identical expected income. The expected minimal income, as well as the expected Gini index, are the same under g_3 and g_4. Yet, the Gini index of the expected income, as well as the minimal expected income, differ.

From a mathematical viewpoint, we note that the average of Choquet integrals is a Choquet integral, and therefore the expected Gini and the expected minimum are representable by a Choquet integral over the states-individuals matrix. By contrast, the minimum of Choquet integrals, or the integral (over individuals) of Choquet integrals (over states) need not be a Choquet integral itself. In other words, the family of Choquet integrals fails to be closed under iterative application.

APPENDIX

Suppose that an inequality measure (under certainty) is represented by a min-of-means functional I_K on F_K that corresponds to a set of measures $C_K \subseteq P_K$. Assume that $p \in P_S$ is an objective probability measure on the state space S. Then the functional

$$I(f) = \alpha E_s[I_K(f_{s\bullet})] + (1-\alpha)\, I_K(E_s[f_{si}]) \qquad \forall f \in F_{S\times K}$$

is a min-of-means functional, and the corresponding set of measures is

$$\left\{ q \in P_{S\times K} \;\middle|\; \begin{array}{c} \exists r^0, r^1, ..., r^{|S|} \in C_K \text{ s.t. } \forall s, i \\ q_{si} = \alpha p_s r_i^s + (1-\alpha)\, p_s r_i^0 \end{array} \right\}.$$

As an example, consider the case of extreme egalitarianism. That is, on the set of individuals K we adopt the set of all probability measures: $C_K = P_K$. Assume that $p \in P_S$ is an objective probability measure as above. Then the functional

$$I(f) = \tfrac{1}{2} E_s[\min_i f_{si}] + \tfrac{1}{2} \min_i E_s[f_{si}] \qquad \forall f \in F_{S \times K}$$

is a min-of-means functional, and its corresponding set of measures (on $S \times K$) is

$$\left\{ q \in P_{S \times K} \;\middle|\; \begin{array}{l} \text{(i) } \sum_i q_{si} = p_s \quad \forall s; \\[2ex] \text{(ii) } \exists r^0 \in P_K \text{ s.t. } q_{si} - \dfrac{p_s r_i^0}{2} \geqslant 0 \quad \forall s, i \end{array} \right\}.$$

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