Ambiguity and the Bayesian Approach

Itzhak Gilboa
Based on papers with Massimo Marinacci, Andy Postlewaite, and David Schmeidler

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The Bayesian Approach

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- Formulation of prior probability on the states space
- Updating of the prior according to Bayes's rule
- Sometimes: also maximizing EU wrt to the probability
  (Not essential, but often axiomatized with the prior.)

Example: Probability of Heart Attack

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- HDL-C (good cholesterol) of 45 mg/dL
- systolic blood pressure is 130.
- What's the probability of a heart attach in the next 10 years?

Estimates from Web Sites

Mayo Clinic	25%
National Cholesterol Education Program	27%
American Heart Association	25% (using additional data)
Medical College of Wisconsin	53%
University of Maryland Heart Center	50%

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- Recovery from economic crisis
- Inflation estimates
- Intentions of world leaders...
- In short, opinions vary
- Is it rational to cling to a unique probability estimate knowing that others have different estimates?

Does Rationality Necessitate Bayesianism?
(See Gilboa, Postlewaite, and Schmeidler, 2009, 2010)

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- Savage's (1954) axiomatic derivation is beautiful, deep, and compelling
- But it may be misleading:
- Completeness is much more convincing for *raw preferences* than for *reasoned choice*

  In fact, it can be circular in cases such as "what's the probability of economic recovery?"

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- But not for large, theoretically-constructed, single-shot state space
- For example: Bush has to decide whether to save Lehman Brothers
  The state will never be observed
  (the decision will determine the information partition, but no single state is ever observable)

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- What’s the probability that all Abordytes and Cyclophines?
- You don’t even know if these are names of enzymes, ancient languages, or Abelian groups...
- This means that a system of axioms that implies that you should have a probability over such events may be less compelling than it appears.

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- Assuming unbounded rationality, one can always define a canonical state space (which describes anything and everything which may be of interest)
- But not a canonical prior over it
- The further back we go, the larger is the state space, and the more arbitrary the prior

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- Hence in some situations it may be less rational to be Bayesian than to be non-Bayesian
- In particular, in all domains of science we use classical statistics, not committing to a single prior over the possible distributions
- Whenever subjective priors vary significantly, one is led to ask, how rational is it for me to cling to my own subjective assessment, if I can't convince others of it?

History

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- Early on there were debates re whether all uncertainty can be quantified
- Shafer (1986) argues that doubts can be found in Bernoulli (1713)
- Revived in the 1920s, with Keynes (1921) and Knight (1921) arguing against the Bayesian approach, and Ramsey (1931) and de Finetti (1931) promoting it

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- By contrast, in economics, it has been applied to the "Grand State Space"

The "Grand State Space"
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- By definition we only observe one state, and never have the chance to repeat the experiment
- The prior over the grand state space is arbitrary
  any information that may be helpful in formalizing it should be incorporated into the description of the decision problem, shifting the problem of prior formulation one stage up

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- It is in the latter applications that we doubt the Bayesian approach

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- This is inconsistent with the Bayesian approach
- Still, many insist on this choice even when the inconsistency and Savage's axioms are explained to them

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- This is reminiscent of (and arguably the reason for) the "home bias": people prefer trading domestic stocks to foreign ones (Epstien and Miao, 2003)

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  If one wishes to be Bayesian, it is easy to adopt a prior in this example (due to symmetry)
- But this is not the case in real life examples of wars, stock market crashes, etc.
- Indeed, Schmeidler's critique was based on the cognitive implausibility of the Bayesian approach, and not on the results of an experiment

Anscombe-Aumann's Framework

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- AA5 non-trivial (there are f, g such that $f \succ g$)

Anscombe-Aumann's Theorem

Theorem

Let $\succsim$ be a preference defined on $\mathcal{F}$. The following conditions are equivalent:

(i) *$\succsim$ satisfies axioms AA.1-AA.5;*

(ii) *there exists a function $u : X \to \mathbb{R}$ and a probability measure $P : \Sigma \to \mathbb{R}$ such that, for all $f, g \in \mathcal{F}$, $f \succsim g$ if and only if*

$$\int_S \left(\sum_{x \in \operatorname{supp} f(s)} u(x) f(s) \right) dP(s) \geq \int_S \left(\sum_{x \in \operatorname{supp} g(s)} u(x) g(s) \right) dP(s).$$

Moreover, P is unique and u is cardinally unique.

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- Consider two states $S = \{1, 2\}$

$$f = (1, 0) \qquad g = (0, 1)$$

- Out of ignorance, $f \sim g$
- But

$$\frac{1}{2}f + \frac{1}{2}g = (0.5, 0.5)$$

provides hedging and is better than both f and g.

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  to the case that all three acts are *pairwise comonotonic*
- i.e., it never happens that one increases between two states while the other decreases
- ... and thus hedging is ruled out

Choquet EU (Schmeidler, 1989)

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  $E \subseteq E'$ implies $v(E) \leq v(E')$

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- $v : \Sigma \to [0, 1]$ is a *capacity* if
  $v(\varnothing) = 0$ and $v(S) = 1$
  $E \subseteq E'$ implies $v(E) \leq v(E')$
- Choquet integral of for $\phi \geq 0$

$$\int \phi \, dv = \int_0^\infty v\left(\{s \in S : \phi\left(s\right) \geq t\}\right) dt$$

Choquet EU (II)
Schmeidler (1986, 1989):

Theorem

Let $\succsim$ be a preference defined on $\mathcal{F}$. The following conditions are equivalent:

(i) *$\succsim$ satisfies axioms AA.1, AA.2, S.3, AA.4, and AA.5;*

(ii) *there exists a function $u : X \to \mathbb{R}$ and a capacity $\nu : \Sigma \to \mathbb{R}$ such that, for all $f, g \in \mathcal{F}$, $f \succsim g$ if and only if*

$$\int_S \left(\sum_{x \in \operatorname{supp} f(s)} u(x) f(s) \right) d\nu(s) \geq \int_S \left(\sum_{x \in \operatorname{supp} g(s)} u(x) g(s) \right) d\nu(s).$$

Moreover, ν is unique and u is cardinally unique.

Another Restriction of Independence

- To be on the safe side, assume only that (GS.3)

$$f \succ g \Rightarrow \alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$$

holds when h is constant.

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- Add the Uncertainty Aversion S.6 (Schmeidler, 1986) axiom:

$$f \sim g \quad \Rightarrow \quad \frac{1}{2}f + \frac{1}{2}g \succsim f, g$$

Maxmin EU (MMEU)
Gilboa-Schmeidler (1989):

Theorem

Let $\succsim$ be a preference defined on $\mathcal{F}$. The following conditions are equivalent:

(i) *$\succsim$ satisfies axioms AA.1, AA.2, GS.3, AA.4, AA.5, and S.6;*

(ii) *there exists a function $u : X \to \mathbb{R}$ and a convex and compact set $C \subseteq \Delta(\Sigma)$ of probability measures such that, for all $f, g \in \mathcal{F}$,*

$$f \succsim g \Leftrightarrow \min_{P \in C} \int_S \left(\sum_{x \in \mathrm{supp}\, f(s)} u(x) f(s) \right) dP(s) \geq \min_{P \in C} \int_S \left(\sum_{x \in \mathrm{supp}\, f(s)} u \right.$$

Moreover, C is unique and u is cardinally unique.

Variational Preferences

- A further restriction of independence by Maccheroni, Marinacci, and Rustichini (2006) yields

$$\min_{p \in C} \left[\int u(f)\, dp + c(p) \right]$$

where c is convex (with $\min c = 0$)

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where c is convex (with $\min c = 0$)

- This generalizes the multiplier preferences used by Hansen and Sargent (2001,...,2008)

$$V(f) = \min_{P \in \Delta(S)} \left\{ \int_S u(f(s))\, dP(s) + \theta R(P \| Q) \right\},$$

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- An alternative (to the MMEU model):
- Take a second-order probability measure, μ, over the possible priors $\Delta(S)$
- Consider some integration of the expected values of an act f, $\int u(f)\, dp$
- But in a non-linear way...

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- Axiomatized by Klibanoff, Marinacci, Mukerji (2005), Nau (2006), Seo (2008)

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- Yet, it is important to perform this test, especially when things cancel out too neatly.