

## Chapter 4: Decisions under Risk

### Problems

In each of the following problems you are asked to choose between two lotteries. A “lottery” gives you certain monetary prizes with given probabilities.

For instance,

|    |         |    |
|----|---------|----|
| A: | \$0     | .5 |
|    | \$1,000 | .5 |

A is a lottery that gives you \$0 with probability 50%, and \$1,000 otherwise.

A “sure” prize will be represented as a lottery with probability 1, say:

|    |       |   |
|----|-------|---|
| B: | \$500 | 1 |
|----|-------|---|

B is a “lottery” that gives you \$500 for sure.

Please denote your preferences between the lotteries by

$$A \prec B \quad \text{or} \quad A \succ B$$

(or  $A \sim B$  if you are indifferent between the two).

**Problem 4.1**

|    |            |    |       |   |
|----|------------|----|-------|---|
| A: | \$0 .5     | B: | \$500 | 1 |
|    | \$1,000 .5 |    |       |   |

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**Problem 4.2**

|    |            |    |       |   |
|----|------------|----|-------|---|
| A: | \$0 .2     | B: | \$500 | 1 |
|    | \$1,000 .8 |    |       |   |

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**Problem 4.3**

|    |            |    |         |    |
|----|------------|----|---------|----|
| A: | \$2,000 .5 | B: | \$1,000 | .5 |
|    | \$4,000 .5 |    | \$5,000 | .5 |

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**Problem 4.4**

|    |            |    |         |    |
|----|------------|----|---------|----|
| A: | \$2,000 .5 | B: | \$1,000 | .4 |
|    | \$4,000 .5 |    | \$5,000 | .6 |

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**Problem 4.5**

|    |         |    |    |         |  |
|----|---------|----|----|---------|--|
| A: | \$0     | .2 | B: | \$3,000 |  |
|    | \$4,000 | .8 |    |         |  |

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**Problem 4.6**

|    |         |    |    |       |    |
|----|---------|----|----|-------|----|
| A: | \$0     | .2 | B: | \$400 | .6 |
|    | \$400   | .6 |    | \$500 | .4 |
|    | \$1,000 | .2 |    |       |    |

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**Problem 4.7**

|    |         |    |    |       |    |
|----|---------|----|----|-------|----|
| A: | \$0     | .1 | B: | \$400 | .5 |
|    | \$400   | .5 |    | \$500 | .5 |
|    | \$1,000 | .4 |    |       |    |

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**Problem 4.8**

|    |            |    |            |
|----|------------|----|------------|
| A: | \$2,000 .2 | B: | \$1,000 .2 |
|    | \$4,000 .2 |    | \$5,000 .2 |
|    | \$6,000 .6 |    | \$6,000 .6 |

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**Problem 4.9**

|    |            |    |             |
|----|------------|----|-------------|
| A: | \$2,000 .2 | B: | \$1,000 .16 |
|    | \$4,000 .2 |    | \$5,000 .24 |
|    | \$6,000 .6 |    | \$6,000 .6  |

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**Problem 4.10**

|    |            |    |             |
|----|------------|----|-------------|
| A: | \$0 .8     | B: | \$0 .75     |
|    | \$4,000 .2 |    | \$3,000 .25 |

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## ***Exercises – Chapter 4***

1. Assume that you are indifferent between getting \$700 and getting \$1000 with probability 80% (and otherwise nothing). Assume also that you are indifferent between getting \$300 and getting **\$700** (not 1000 this time!) with probability 60% (and otherwise nothing). Consider lottery A, which gives you \$1000 with probability  $2/3$  (and otherwise nothing), and lottery B, which gives you a 50%-50% bet between \$300 and \$700 dollar. If you follow von-Neumann-Morgenstern's theory, you should:
  - a. Prefer A to B
  - b. Prefer B to A
  - c. Be indifferent between A and B
  - d. One cannot tell based on the data.
  
2. Mary likes the von Neumann Morgenstern's axioms and she would like to make decisions in accordance with these axioms. By careful introspection, she has decided that she would be indifferent between

\$400 for sure and a 50% of obtaining \$1,000 (otherwise – nothing);

and also between

\$600 for sure and a 80% of obtaining \$1,000 (otherwise – nothing).

Mary is offered a bet among (\$0, \$400, \$600, \$1,000) with equal chances (25% each) for a cost of \$400. Should she prefer the bet or should she prefer to keep her \$400?

3. A state lottery sells tickets for a cost of \$1 each. The ticket has a probability of  $1/(2,400,000)$  of winning \$1,000,000, and otherwise – nothing.
  - a. What is the expected profit of the state from each ticket sold?
  - b. In the hope of increasing profits, the state considers to increase the award to \$2,000,000 and to reduce the probability of winning to  $1/(4,800,000)$ . A statistician said that it's not worth the trouble, because the expected profit remains precisely the same. What do you think?
4. It is often argued that the value function in Kahneman and Tversky's Prospect Theory is convex in the domain of losses, that is, that individuals behave in a risk loving way when it comes to losses. How can this be reconciled with the fact that people buy insurance (where premia exceed expected losses)?