Questions in Decision Theory

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History

- **Pascal and Bernoulli**
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- Ramsey and deFinetti
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- von Morgenstern-Neumann
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- Savage
- Anscombe-Aumann
The Bible (Savage, 1954)

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- **P2** $f_{A^c}^h \succeq g_{A^c}^h$ iff $f_{A^c}^{h'} \succeq g_{A^c}^{h'}$
- **P3** $x \succeq y$ iff $f_{A}^x \succeq f_{A}^y$
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- \( P4 \) \( y^x_A \sim y^x_B \) iff \( w^z_A \sim w^z_B \)
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- **P5** $\exists f \succ g$
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- **P5** \( \exists f \succ g \)
- **P6** \( f \succ g \) \( \exists \) a partition of \( S \), \( \{ A_1, \ldots, A_n \} \) \( f_{A_i}^h \succ g \) and \( f \succ g_{A_i}^h \)
Savage's Theorem

Assume that $X$ is finite. Then $\succsim$ satisfies P1-P6 if and only if there exist a non-atomic finitely additive probability measure $\mu$ on $S (= (S, 2^S))$ and a non-constant function $u : X \to \mathbb{R}$ such that, for every $f, g \in F$

$$f \succsim g \iff \int_S u(f(s))d\mu(s) \geq \int_S u(g(s))d\mu(s)$$

Furthermore, in this case $\mu$ is unique, and $u$ is unique up to positive linear transformations.
Decision Theory at a Crossroad

- Accuracy vs. beauty/generality
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- Method: experiments, axioms, neurological data?
- Goal: theoretical models or applied decisions?
- Descriptive or normative?
Main Questions

- Rationality
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- Rules and analogies
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- Utility
- Rules and analogies
- Group decisions
Rationality

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- An even more subjective view: which consistency?
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- Weaknesses (?): subjective, empirical, not monotonic in intelligence
- Defense
Objective and Subjective Rationality

- A decision maker is defined by two relations $(\succeq^*, \succeq^\wedge)$
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- $\simeq^*$ – can convince “any reasonable decision maker” that it is right
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- $\succeq^*$ – can convince “any reasonable decision maker” that it is right
- $\succeq^\wedge$ – cannot be convinced that it is wrong
- Clearly, $\succeq^* \subset \succeq^\wedge$
Classical and Bayesian Statistics

- Classical: attempts to be objective, no intuition
- Bayesian: attempts to incorporate intuition and hunches

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Bayesian – for making a decision (for oneself)
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- What is the probability of
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  - A coin coming up Head?
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- A car being stolen?
- A surgery succeeding?
- A war erupting?
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Subjective probability

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- The Bayesian approach is good at representing knowledge, poor at representing ignorance
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Objective probabilities

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- Rule-based approaches: logit
- Case-based approaches: empirical similarity
- But none extends to the cases of wars, stock market crashes...
Alternatives to the Bayesian approach

- Schmeidler (1989): non-additive probabilities (capacities)
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Alternatives to the Bayesian approach

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- Integration by Choquet’s integral
- Maxmin EU: there exists a set of probabilities $C$ such that

$$V(f) = \min_{P \in C} \int_S u(f(s)) \, dP(s)$$
Other multiple-priors models

- Nau, Klibanoff-Marinacci-Mukerji: “smooth preferences”

\[ \varphi : \mathbb{R} \rightarrow \mathbb{R} \]

\[ \int_{\Delta(S)} \varphi \left( \int u(f) \, dp \right) \, d\mu \]
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  \[ \int_{\Delta(S)} \varphi \left( \int u(f) \, dp \right) \, d\mu \]

- Maccheroni-Marinacci-Rustichini: “variational preferences”
  \[ V(f) = \min_{P \in \Delta(S)} \left\{ \int_S u(f(s)) \, dP(s) + c(P) \right\} \]
Incomplete relation

• Bewley:

\[ f \succ g \text{ iff } \forall p \in C \int_S u(f(s)) \, dP(s) > \int_S u(g(s)) \, dP(s) \]
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- Fits the “objective rationality” notion
Incomplete relation

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- Fits the “objective rationality” notion

- Can be combined with the maxmin criterion as “subjective rationality”
Utility

- What is utility and how is it related to well-being or happiness?
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- Measurement of well-being and its relation to money
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- All happy families...?
Rules and analogies

- In the context of probability
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- In the context of probability
- Statistics
Rules and analogies

- In the context of probability
- Statistics
- Moral argumentation
Rules and analogies

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- Moral argumentation
- Recent model unifying the two, as well as Bayesian
Group decisions

- Do groups make better decisions than do individuals?
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- “Truth wins” vs. risk/uncertainty aversion
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- Aggregation of opinions/judgment aggregation