Rationality and the Bayesian Paradigm

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Rationality

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency
- An even more subjective view: which consistency?
- Rationality as robustness
- Weaknesses (?): subjective, empirical, not monotonic in intelligence
- Defense

Objectivity and Subjectivity

- Anscombe-Aumann
- Schmeidler's example
- Objectivity as second-order subjectivity

Objective and Subjective Rationality

- A decision maker is defined by two relations $(\succsim^*, \succsim^\wedge)$
- $\succsim^*$ – can convince "any reasonable decision maker" that it is right
- $\succsim^\wedge$ – cannot be convinced that it is wrong
- Clearly, $\succsim^* \subset \succsim^\wedge$

The Bayesian Approach

- Formulate state space
- All uncertainty resolved by the state
- Formulate a prior probability
- Update by Bayes's rule

Classical and Bayesian Statistics

- Classical: attempts to be objective, no intuition
- Bayesian: attempts to incorporate intuition and hunches
- Classical – for making a point (to others)
- Bayesian – for making a decision (for oneself)

Rationality and Bayesianism

- Pascal and Bernoulli
- Ramsey and de Finetti
- von Neumann-Morgenstern
- Savage
- Anscombe-Aumann

The Bible (Savage, 1954)

- $F = X^S = \{f \mid f : S \to X\}$
- **P1** $\succsim$ is a weak order
- **P2** $f^h_{A^c} \succsim g^h_{A^c}$ iff $f^{h'}_{A^c} \succsim g^{h'}_{A^c}$
- **P3** $x \succsim y$ iff $f^x_A \succsim f^y_A$
- **P4** $y^x_A \succsim y^x_B$ iff $w^z_A \succsim w^z_B$
- **P5** $\exists\ f \succ g$
- **P6** $f \succ g$ $\exists$ a partition of S, $\{A_1, ..., A_n\}$ $f^h_{A_i} \succ g$ and $f \succ g^h_{A_i}$

Savage's Theorem

- Assume that X is finite. Then $\succsim$ satisfies P1-P6 if and only if there exist a non-atomic finitely additive probability measure μ on S $(=(S, 2^S))$ and a non-constant function $u : X \to \mathbb{R}$ such that, for every $f, g \in F$

$$f \succsim g \quad \text{iff} \quad \int_S u(f(s))d\mu(s) \geq \int_S u(g(s))d\mu(s)$$

Furthermore, in this case μ is unique, and u is unique up to positive linear transformations.

What's in a State?

- de Finetti, Harsanyi, Aumann
- Newcombe: also causal relationships
- A problem for a behavioral derivation
- Where would the probability come from?

Probability – Whence?

- What is the probability of
- A coin coming up Head?
- A car being stolen?
- A surgery succeeding?
- A war erupting?

Subjective Probability

- Normative interpretation: completeness?
- If it's so rational, why isn't it objective?
- Are all Arbodytes Cyclophines?
- The Bayesian approach is good at representing knowledge, poor at representing ignorance

Objective Probabilities

- Exist in simple cases (iid)
- Can be defined with identicality, as long as causal independence is retained
- Rule-based approaches: logit
- Case-based approaches: empirical similarity
- But none extends to the cases of wars, stock market crashes...

Alternatives to the Bayesian Approach

- Schmeidler (1989): non-additive probabilities (capacities)
- Integration by Choquet's integral
- Maxmin EU: there exists a set of probabilities C such that

$$V(f) = \min_{P \in C} \int_S u(f(s))\, dP(s)$$

Other Multiple-Priors Models

- Nau, Klibanoff-Marinacci-Mukerji: "smooth preferences"

$$\varphi : \mathbb{R} \rightarrow \mathbb{R}$$

$$\int_{\Delta(S)} \varphi \left(\int u(f)\, dp \right) d\mu$$

- Maccheroni-Marinacci-Rustichini: "variational preferences"

$$V(f) = \min_{P \in \Delta(S)} \left\{ \int_S u(f(s))\, dP(s) + c(P) \right\}$$

Incomplete Preferences

- Bewley:

$$\begin{aligned} f &\succ g \\ & iff \\ \forall p &\in C \\ \int_S u(f(s))\, dP(s) &> \int_S u(g(s))\, dP(s) \end{aligned}$$

- Fits the "objective rationality" notion
- Can be combined with the maxmin criterion as "subjective rationality"