Objective and Subjective Rationality

in a Multiple Prior Model

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The Binary Relation

Raw preferences vs. reasoned choice

The Role of Axioms

description of choice vs. reasoning templates

BUILDING THE PREFERENCES

A policymaker seeks the advice of experts for an important decision.

The policy maker wishes to know what her policy should be. That is, she constructs her preferences in as rational a way as she can.

The axiomatic models of decision under uncertainty, using *one* single binary relation, are too austere to describe how a decisiom maker (DM) generates her preferences, or how she might change her mind.

We consider two *binary relations as primitives, rather than one.*

OBJECTIVE RATIONALITY

$\succsim^*$ denotes preferences that are *rational in an objective sense*: The DM can be convinced that, according to her objectives, f is least as desirable as g.

Objective rationality consists of those decisions that the DM was not necessarily inclined to make prior to meeting her advisor, but ends up making.

Unfortunately, in many decision problems under uncertainty, $\succsim^*$ may fail to be complete.

SUBJECTIVE RATIONALITY

$\succsim^{\circ}$ will reflect preferences that are *rational in a subjective sense*: Out of f and g, the DM will choose f (or f and g). She might not be able to fully defend her choice of f over g, but she will not be embarrassed by an analysis of her choices. She will also not be convinced that she should have made different ones.

Subjective rationality allows for various gut feelings that exist before meeting the advisor, and still remain after the meeting.

$\succsim^{\circ}$ is complete almost by definition.

TWO RELATIONS

- In/completeness of beliefs/tastes
  - Ghirardato, Maccheroni, Marinacci (JET 2004)
  - Nehring (JET 2008)
- Psychological and revealed preferences
  - Mandler (GEB 2005)
  - Danan (2006)
- Status quo bias completion
  - Masatlioglu and Ok (JET 2005)

SETUP

- (S, Σ) a measurable space of *states of the world*;

- X a convex set of *consequences*;

- Δ the set of *probabilities* on Σ (with the weak* topology);

- $\mathcal{F}$ the set of all *acts*: simple measurable functions from S to X;

- $\succsim^*$ and $\succsim^\circ$ two binary relations on $\mathcal{F}$.

BASIC CONDITIONS

We make some common assumptions on both on $\succsim^*$ and $\succsim^\circ$...

BC The following conditions are satisfied:

(a) $\succsim$ is a preorder.

(b) If $f, g \in \mathcal{F}$ and $f(s) \succsim g(s)$ for all $s \in S$, then $f \succsim g$.

(c) For all $f, g, h \in \mathcal{F}$, the sets $\{\lambda \in [0,1] : \lambda f + (1-\lambda)g \succsim h\}$ and $\{\lambda \in [0,1] : h \succsim \lambda f + (1-\lambda)g\}$ are closed in $[0,1]$.

(d) There exist $f, g \in \mathcal{F}$ such that $f \succ g$.

COMPLETENESS & INDEPENDENCE

C-Completeness For every $x, y \in X$, $x \succsim^* y$ or $y \succsim^* x$.

Independence If $f, g, h \in \mathcal{F}$, and $\alpha \in (0, 1)$, then

$$f \succsim^* g \Longleftrightarrow \alpha f + (1-\alpha)h \succsim^* \alpha g + (1-\alpha)h.$$

Completeness For every $f, g \in \mathcal{F}$, $f \succsim^\circ g$ or $g \succsim^\circ f$.

C-Independence If $f, g \in \mathcal{F}$, $x \in X$, and $\alpha \in (0, 1)$, then

$$f \succsim^\circ g \Longleftrightarrow \alpha f + (1-\alpha)x \succsim^\circ \alpha g + (1-\alpha)x.$$

Uncertainty Aversion If $f, g \in \mathcal{F}$, and $f \sim^\circ g$, then $\frac{1}{2}f + \frac{1}{2}g \succsim^\circ g$.

UNANIMITY

Theorem 1 *TFAE:*

(i) $\succsim^*$ *satisfies BC, C-Completeness, and Independence;*

(ii) there exist a non-empty closed and convex set C_* *of probabilities on* Σ *and a non-constant affine* $u_* : X \to \mathbb{R}$ *such that, for* $f, g \in \mathcal{F}$

$$f \succsim^* g \iff \int u_*(f)\, dp \geq \int u_*(g)\, dp \quad \forall p \in C_*$$

C_* *is unique and* u_* *is cardinally unique.*

À la Bewley (DEF 2002), see also Ghirardato, Maccheroni, Marinacci, and Siniscalchi (ECM 2003), Girotto and Holzer (JMP 2005), Baucells and Shapley (GEB 2008), Ok, Ortoleva, and Riella (2008).

MAXMIN

Theorem 2 (Gilboa and Schmeidler, JME 1989) *TFAE:*

(i) $\succsim^\circ$ *satisfies BC, Completeness, C-Independence, and Uncertainty Aversion;*

(ii) there exist a non-empty closed and convex set $C_\circ$ *of probabilities on* Σ *and a non-constant affine* $u_\circ : X \to \mathbb{R}$ *such that, for* $f, g \in \mathcal{F}$

$$f \succsim^\circ g \iff \min_{p \in C_\circ} \int u_\circ(f)\, dp \geq \min_{p \in C_\circ} \int u_\circ(g)\, dp$$

$C_\circ$ *is unique and* $u_\circ$ *is cardinally unique.*

CONSISTENCY AND CAUTION

Consistency If $f, g \in \mathcal{F}$ and $f \succsim^* g$, then $f \succsim^\circ g$.

Caution If $f \in \mathcal{F}$, $x \in X$, and $f \not\succsim^* x$, then $x \succsim^\circ f$.

Default to Certainty If $f \in \mathcal{F}$, $x \in X$, and $f \not\succsim^* x$, then $x \succ^\circ f$.

THE RESULT

Theorem 3 *TFAE:*

(i) $(\succsim^*, \succsim^\circ)$ *satisfy the conditions (i) of Theorems 1 and 2, resp., jointly, they satisfy Consistency and Caution;*

(ii) There exist a non-empty closed and convex set C *of probabilities on* Σ *and a non-constant affine* $u : X \to \mathbb{R}$ *such that, for* $f, g \in \mathcal{F}$

$$f \succsim^* g \Longleftrightarrow \int u(f)\, dp \geq \int u(g)\, dp \quad \forall p \in C$$

and

$$f \succsim^\circ g \Longleftrightarrow \min_{p \in C} \int u(f)\, dp \geq \min_{p \in C} \int u(g)\, dp$$

C *is unique and* u *is cardinally unique.*

DEFAULT TO CERTAINTY

Theorem 4 *TFAE:*

(i) $\succsim^*$ *satisfies BC, C-Completeness, and Independence,*
$\succsim^\circ$ *is a continuous weak order,*
jointly, they satisfy Consistency and Default to Certainty;

(ii) There exist a non-empty closed and convex set C *of probabilities on* Σ *and a non-constant affine* $u : X \to \mathbb{R}$ *such that, for* $f, g \in \mathcal{F}$

$$f \succsim^* g \Longleftrightarrow \int u(f)\, dp \geq \int u(g)\, dp \quad \forall p \in C$$

and

$$f \succsim^\circ g \Longleftrightarrow \min_{p \in C} \int u(f)\, dp \geq \min_{p \in C} \int u(g)\, dp$$

C *is unique and* u *is cardinally unique.*

WRAPPING UP

- Rationality.
- Objectivity/Subjectivity.
- Axioms.