Consumer Constrained Imitation

Itzhak Gilboa, Andrew Postlewaite, and David Schmeidler

April 16, 2015

Common critiques of the neoclassical model

- The neoclassical model is not supposed to reflect a reasoning process
- Yet, sometimes it seems suspiciously far removed from the way people think
- Example: caffe latte and a car
- Evidence: Binkley and Bejnarowicz (2003) “Consumer Price Awareness in Food Shopping: The Case of Quantity Surcharges,” Journal of Retailing.

The Affluent Society

- Galbraith (1958)
- Consider a graduate student getting a first job
- Poor consumers might be closer to the neoclassical model
  - Can't afford the luxury of not thinking
  - Have a much smaller space of solutions to consider
  - Have well-defined preferences when in the domain of necessity

The Complexity of the Consumer Problem

- Consider a problem $P = \left\langle n, (p_i)_{i \leq n}, I, u \right\rangle$ whose input is:
  - $n \geq 1$ – the number of *products*;
  - $p_i \in \mathbb{Z}_+$ is the *price* of product $i \leq n$;
  - $I \in \mathbb{Z}_+$ is the consumer's *income*; and
  - $u : \mathbb{Z}_+^n \to \mathbb{R}$ is the consumer's *utility* function given as a formula (with "$+$", "$\cdot$"...)
- *Consumer Problem*: Given $P = \left\langle n, (p_i)_{i \leq n}, I, u \right\rangle$ and $\bar{u}$, can utility $\bar{u}$ be obtained in P?

The Complexity Result

Proposition

The Consumer Problem is NP-Complete.

- Some variables are integer valued
  - As housing, cars, education...
- The utility function is part of the input
  - Reflecting the assumption that a product's attributes are encapsulated in u
- Still, econ 101 argues that households solve an NPC problem.

So What Do Households Do?

- Top-down approach: dividing budgets among categories
  - This is similar to the way data are organized
- We will focus on the top level
  - Which can be repeated recursively
- How are choices made, without “yet” knowing the marginal utility of expense on each category?

Imitation

- Household do what others do, and what they did in the past
  - Galbraith: “emulation”
- This could be due to:
  - Social learning: private signals and/or cognitive free-riding
  - Conformism
  - Ill-defined preferences once necessities are satisfied
- We will not attempt to distinguish among these here

Constrained Case-Based Decisions

- Given a database of points in the budget proportions simplex, the household chooses a similarity-weighted average
  - More similar households get more weight
  - As does the household's own past decisions
- However, the above is constrained to satisfy some rules of thumb
  - Which can also indicate the household's intrinsic preferences

Formally

- Income I
- Expenditures $E_1, ..., E_n$

$$E_1 + ... + E_n = I$$

- Budget shares

$$z_i = \frac{E_i}{I}$$

- "Quantity" of category i

$$\frac{z_i I}{p_i}$$

Pure Imitation

- A database of past choices

$$D = \left(\left(x_{1t}, ..., x_{mt} \right), \left(z_{1t}, ..., z_{nt} \right) \right)_{t=1}^{T}$$

- A similarity function $s : C \longrightarrow \mathbb{R}_{++}$ (of case $\left(x_{1t}, ..., x_{mt} \right)$ to the current one)
- The household chooses

$$\frac{\sum_{t \leq T} s(x_t) z_t}{\sum_{t \leq T} s\left(x_t\right)}$$

But There Are Constraints

- Examples:
  - "housing expenditure *should not exceed* 40% of the budget"
  - "savings *should be at least* 25% of the budget"
- For some set A there exists a collection

$$\{ f_\alpha (z) \geq c_\alpha \mid \alpha \in A \}$$

where f_α is a linear function and $c_\alpha \in \mathbb{R}$.

Extreme Case

- The constraints can leave very little room for imitation:
- Consider the constraints

$$\begin{aligned} z_i &\leq \beta_i \\ z_i &\geq \beta_i \end{aligned}$$

  - where $\beta = (\beta_1, ..., \beta_n)$ is a vector in the simplex.
- Equivalent to maximizing the Cobb-Douglas function

$$u\left(\left(\frac{z_i I}{p_i}\right)_{i=1}^{n}\right) = \prod_{i=1}^{n}\left(\frac{z_i I}{p_i}\right)^{\beta_i}$$

Constrained Imitation

- A database
$$D = \left(\left(x_{1t}, \ldots, x_{mt}\right), \left(z_{1t}, \ldots, z_{nt}\right)\right)_{t=1}^{T}$$
- A similarity function $s : C \to \mathbb{R}_{++}$
- Constraints
$$F \equiv \left\{ f_{\alpha}\left(z\right) \geq c_{\alpha} \mid \alpha \in A \right\}$$
such that
$$Z \equiv \cap_{\alpha \in A} \left\{ z \in \Delta(\Omega) \mid f_{\alpha}\left(z\right) \geq c_{\alpha} \right\} \neq \varnothing$$
- Choose
$$\frac{\sum_{t \leq T} s(x_t) y\left(z_t\right)}{\sum_{t \leq T} s\left(x_t\right)}$$
where $y\left(z_t\right)$ is the closest point to z in Z.

The (Neo-)Classical Model

- Formally can be embedded in the above:

Proposition

Let there be given a concave utility function u. Then there exists a (typically infinite) set of constraints $F \equiv \{ f_\alpha (z) \geq c_\alpha \mid \alpha \in A \}$ such that, for every similarity function s and every database D every optimal solution to $P(F, s, D)$ defines a maximizer of u (with quantities $z_i I / p_i$).

- Though the constraints aren't necessarily intuitive

Axiomatic Foundations

- $\Omega = \{1, ..., n\}$ – *categories*, $n \geq 3$
- $C = X \times \Delta(\Omega)$ – a non-empty set of *cases*
  - with X being a subset of $\mathbb{R}^k$ for some $k \geq 0$
- A *database*: $D \in C^r$ for $r \geq 1$
- $C^* = \cup_{r\geq 1} C^r$ – all databases
- Concatenation of databases $D = (c_1, ..., c_r) \in C^r$ and $E = (c'_1, ..., c'_t) \in C^t$

$$D \circ E = (c_1, ..., c_r, c'_1, ..., c'_t) \in C^{r+t}$$

- For $D \in C^r$ and a permutation $\pi \in \Pi_r$, let $\pi D \in C^r$ be the permuted database.

Axioms

- Behavior: $Y : C^* \to \Delta(\Omega)$
- **Invariance**: For every $r \geq 1$, every $D \in C^r$, and every permutation $\pi \in \Pi_r$, $y(D) = y(\pi D)$.
- **Concatenation**: For every $D, E \in C^*$, $y(D \circ E) = \lambda y(D) + (1 - \lambda) y(E)$ for some $\lambda \in (0, 1)$.

Similarity-Weighted Averaging

- The following result appeared in Billot, Gilboa, Samet, and Schmeidler (2005):

Theorem

Let there be given a function $Y : C^* \to \Delta(\Omega)$. The following are equivalent:
(i) Y satisfies the Invariance axiom, the Concatenation axiom, and not all $\{Y(D)\}_{D \in C^*}$ are collinear;
(ii) There exists a function $y : C \to \Delta(\Omega)$, where not all $\{y(c)\}_{c \in C}$ are collinear, and a function $s : C \to \mathbb{R}_{++}$ such that, for every $r \geq 1$ and every $D = (c_1, ..., c_r) \in C^r$,

$$Y(D) = \frac{\sum_{j \leq r} s(c_j) y(c_j)}{\sum_{j \leq r} s(c_j)}. \qquad (*)$$

Moreover, in this case the function y is unique, and the function s is unique up to multiplication by a positive number.

Axioms – Con't

- By the theorem, there is $y : C \to \Delta(\Omega)$ which determines Y via s-averaging
- Assume also
- **A3 Independence**: For all $x, x' \in X$, and all $z \in \Delta(\Omega)$, $y((x,z)) = y((x',z))$
- **A4 Distance**: For all $z \in \Delta(\Omega)$ and all $z', z'' \in \mathrm{Im}(y)$, if $z' = y(z)$ and $z'' \neq z'$ then $\|z' - z\| < \|z'' - z\|$.

(Where Im denotes image of a function.)

Convex Feasible Set

Theorem

The following are equivalent:
(i) The function y satisfies A3 and A4;
(ii) There exists a set of constraints

$$\{\, f_\alpha(z) \geq c_\alpha \mid \alpha \in A \,\}$$

where f_α are linear functions and $c_\alpha \in \mathbb{R}$ such that

$$Z \equiv \cap_{\alpha \in A} \{\, z \in \Delta(\Omega) \mid f_\alpha(z) \geq c_\alpha \,\} \neq \varnothing$$

and, for all $x \in X$ and all $z \in \Delta(\Omega)$, $y(z) = y((x, z))$ is the closest point to z in Z.
Further, in this case the set Z is unique and it is the image of y.

Putting It All Together

Theorem

Y is not collinear, it satisfies A1, A2, and the resulting y satisfied A3 and A4 IFF
There exists a $s : C \to \mathbb{R}_{++}$ and a set of constraints

$$\{ f_\alpha(z) \geq c_\alpha \mid \alpha \in A \}$$

such that $Z \equiv \cap_{\alpha \in A} \{ z \in \Delta(\Omega) \mid f_\alpha(z) \geq c_\alpha \}$ is a non-empty set which is not contained in an interval, and, for every $r \geq 1$ and every $D = (c_1, ..., c_r) \in C^r$, $Y(D)$ is

$$\frac{\sum_{j \leq r} s(c_j) y(c_j)}{\sum_{j \leq r} s(c_j)}$$

where, for each $c = (x, z)$, $y(c)$ is the closest point to z in Z.
Furthermore, in this case the set Z is unique and the similarity function is unique up to multiplication by a positive constant.

Conclusion

- Mental Accounting
  - May result from a budget allocation DAG that is not a tree
- Causal Accounts
  - Different stories are compatible with the model
- Normative Questions
  - How do we judge allocations?
  - Indeed, how do we define Pareto optimality with affluent households? Are people better off when they can buy things they never thought they needed?
  - Normative economics should be conducted within cognitively plausible models.