Authorization Decisions

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- Existence of market (medication)
- A specific transaction (construction project)
- Granting property rights (tenure)

Rational Choice Approaches

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- Again... (observability; equilibrium selection)

Vast literature in organization theory

Efficient production: Smith, 1776, Marx, 1867, and Durkheim, 1893

Well-tuned machine: Taylor, 1911, Follett, 1918, Fayol, 1919

Bureaus as production units: Niskanen, 1971, 1975

Decomposing the organization: Weber, 1921, 1924 (on authority and bureaucracy)

For the state: Buchanan and Tullock, 1962

Decision making: March and Simon, 1958 (satisficing)

Other metaphors: organisms, brains, cultures, political systems...

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- Highlighting the notions of consistency
- – with past decisions and with regulations
- – and the power of bureaucracy

Model

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- A history $H \subset C$
- A *decision correspondence*

$$f : \{ (H, p) \mid p \in P, H \in \mathcal{H}, p \notin H_P \} \twoheadrightarrow \{0, 1\}$$

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- Define the decision correspondence

$$f_{w_0,w_1}(H, p) = \arg\max_{d\in\{0,1\}} W_{w_0,w_1}(H, p, d)$$

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- **Axiom 3 (Monotonicity):** IF $d \in f(H,p)$, THEN $d \in f(H \cup \{(q,d)\}, p)$.

Representation

Theorem

A decision correspondence f satisfies Axioms 1-3 if and only if there are relevance functions $w_0, w_1 : P \times P \rightarrow \mathbb{R}_+$ such that $f = f_{w_0,w_1}$.

Rules as Constraints

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- Rule-constrained decisions: for H, a set of rules R, and a problem p,

$$f_{w_0,w_1}(H, R, p) = \begin{cases} d & \textit{if} \quad p \in R(d) \backslash R(1 \\ \arg\max_{d\in\{0,1\}} W_{w_0,w_1}(H, p, d) & \textit{otherwise} \end{cases}$$

Language of Regulations

Binary *attributes* $a_1, ..., a_m,\ a_j : P \rightarrow \{0, 1\}$

A *regulation* (J, b, d) with $J \subset \{1, ..., m\}$, $J \neq \varnothing$, $b : J \rightarrow \{0, 1\}$ $d \in \{0, 1\}$.

It is the rule $r = (D(J, b), d)$ where

$$D(J, b) = \{\, p \in P \mid a_j(p) = b(j) \quad \forall j \in J \,\}.$$

Imposing Decisions by Regulations

Proposition

Let there be given a number of attributes m*, a set of regulations* $R = \{(J_i, b_i, d_i)\}_{i=1}^{n}$*, a problem* p *and a decision* d*. There exists a polynomial-time algorithm that finds out whether* R *is consistent, whether* $p \in R(0), R(1)$*, and, if* R *is consistent and* $p \notin R(0), R(1)$*, finds a regulation* $(J_{n+1}, b_{n+1}, d_{n+1})$*, such that* $d_{n+1} = d$*,* $p \in D(J_{n+1}, b_{n+1})$ *and* $R' = \{(J_i, b_i, d_i)\}_{i=1}^{n+1}$ *is consistent.*

The Complexity of Minimal Regulations

Theorem

Let there be given a number of attributes m, a set of regulations $R = \{(J_i, b_i, d_i)\}_{i=1}^{n}$, *a problem p, a decision d, and a number* $k \geq 1$ *such that R is consistent and* $p \notin R(0), R(1)$. *Finding whether there exists a regulation* $(J_{n+1}, b_{n+1}, d_{n+1})$ *such that* $d_{n+1} = d$, $p \in D(J_{n+1}, b_{n+1})$, $R' = \{(J_i, b_i, d_i)\}_{i=1}^{n+1}$ *is consistent, and* $|J_{n+1}| \leq k$ *is NP-Complete.*

Regulations as Mega-Cases

- Rather than constraints, regulations are “more relevant” precedences
- Can explain why some regulations are enforced and others are ignored
- (Regulations as the bed of the river)

Bureaucracy

Hierarchy of languages and of decisions

A decision $d \in \{0, 1\}$ at the top level

And $d_1, ..., d_l \in \{0, 1\}$ at the lower level

An implementation function

$$\varphi : \{0, 1\}^l \rightarrow \{0, 1\}$$

For example,

$$d = d_1 \vee ... \vee d_l$$

The Complexity of Implementation

Proposition

Given a problem $p \in P$*, a decision* $d \in \{0, 1\}$*, decision variables* $d_1, \ldots, d_l$*, and an implementation function* φ*, finding whether the decision* d *can be implemented in* p *is NP-Complete.*

Budgets

Tasks $1, ..., \tau$, with expenses e_i

Budgets $B_1, ...B_s$

$L_{ij} \in \{0, 1\}$ denoting whether task i be funded by budget j

The allocation $A_{ij} \in \{0, 1\}$ is *consistent* if

$$A_{ij} \leq L_{ij} \qquad \forall i, j$$

$$\sum_{j \leq s} A_{ij} = 1 \qquad \forall i$$

and

$$\sum_{i \leq \tau} A_{ij} e_i \leq B_j \qquad \forall j$$

Finding a Consistent Allocation

Proposition

Given expenses $(e_i)_{i \le \tau}$, $(B_j)_{j \le s}$, *and* $(L_{ij})_{i \le \tau, j \le s}$ *finding whether there exists a consistent allocation* $(A_{ij})_{i \le \tau, j \le s}$ *is NP-Complete.*

Implications of Complexity

- Bureaucracies face NP-Hard problems
- Hence they can pretend that a solution does not exist
- Hence they might stick to known solutions (even if they intend to implement new decisions)

Conclusion

- A theory of decisions without a utility function
- Needs to be incorporated with utility-maximizing behavior of agents
- In general, CBDT has both act and result
- In prediction – only results
- In this model (as in court decisions) – only acts.