DECISION THEORY

BIASES and RATIONALITY –

Some general comments

Decision Theory

- Formidable foundations
  - Probability and reasoning about the future
  - Rational decision making
  - Deeply rooted in the Enlightenment
- Major leaps in the mid-20th
  - Decision theory, game theory
  - Probability and statistics
  - Microeconomics
  - Operations Research

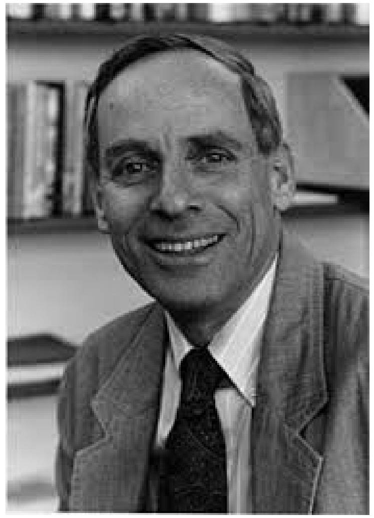
Psychology

Daniel Kahneman

(b. 1934)

Amos Tversky

(1937-1996)



The Project

Daniel Kahneman and Amos Tversky more or less showed that no assumption of rationality holds.

Tversky: "Show me the axiom and I'll design the experiment that refutes it"

(Many of the examples we discuss here are theirs)

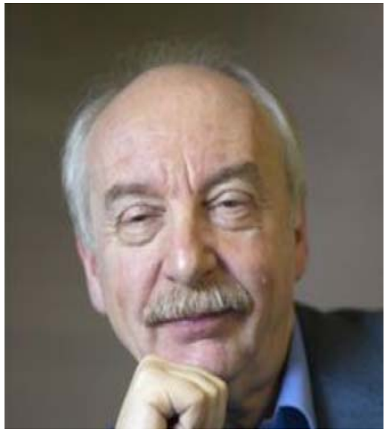
Questions about K-T's Project

How robust are the findings?

– Gigerenzer's Critique

How relevant are they to economics?

… We will discuss these throughout the course



Gerd Gigerenzer (b. 1947)

What do we do with conflicts?

How should we react when theory and evidence are at odds?

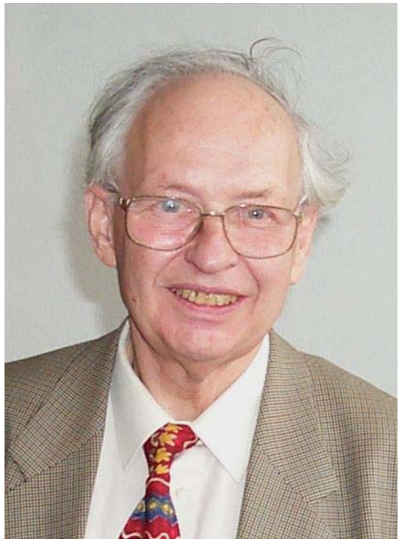
Two possible reactions:

- Bring theory closer to reality (a better descriptive theory)
  - This is what science does in general
  - This is the direction that Behavioral Economics took
- Bring reality closer to theory (use the theory as a normative one)
  - Recall that we are in the social sciences
  - We can change our behavior

“Bounded” Rationality?

Selten once said,

“A normative theory that tells you to run 100m in 4’’ isn’t very useful.”



Reinhard Selten (1930-2016)

Is a violation of the theory…

– Robust?
  - Stable across experiments
  - Has external validity
– Relevant?
  - Occurs in economically relevant problems
  - Has ecological validity (for economics)
– Rational?
  - Cannot, or will not be easily fixed

BIASES

Framing Effects

Operation?

A 65-year old relative of yours suffers from a serious disease. It makes her life miserable, but does not pose an immediate risk to her life. She can go through an operation that, if successful, will cure her. However, the operation is risky. (A: 30% of the patients undergoing it die. B: 70% of the patients undergoing it survive.) Would you recommend that she undergoes it?

The rare disease problem

Imagine that the U.S. is preparing for the outbreak of a rare disease, which is expected to kill 600 people. Two alternative programs to combat the disease are proposed. Assume the exact scientific estimate of the consequences of the programs are:

(a) Exactly 200 people will be saved

(b) There is a probability of $\frac{1}{3}$ that 600 people will be saved and a probability of $\frac{2}{3}$ that no-one will be saved

Alternatively, suppose you choose between:

(c) Exactly 400 people will die

(d) There is a probability of $\frac{1}{3}$ that no-one will die and a probability of $\frac{2}{3}$ that 600 people will die

Majority prefers (a) over (b), but (d) over (c)!!

Reference

The framing of decisions and the psychology of choice

Amos Tversky, Daniel Kahneman

Science 30 : Vol. 211, Issue 4481 (Jan 1981), pp. 453-458

Abstract

The psychological principles that govern the perception of decision problems and the evaluation of probabilities and outcomes produce predictable shifts of preference when the same problem is framed in different ways. Reversals of preference are demonstrated in choices regarding monetary outcomes, both hypothetical and real, and in questions pertaining to the loss of human lives. The effects of frames on preferences are compared to the effects of perspectives on perceptual appearance. The dependence of preferences on the formulation of decision problems is a significant concern for the theory of rational choice.

Reference

Choices, Values and Frames

Daniel Kahneman, Amos Tversky

American Psychologist, Vol. 39 (April 1984), pp. 341–350

Abstract

Discusses the cognitive and the psychophysical determinants of choice in risky and riskless contexts. The psychophysics of value induce risk aversion in the domain of gains and risk seeking in the domain of losses. The psychophysics of chance induce overweighting of sure things and of improbable events, relative to events of moderate probability. Decision problems can be described or framed in multiple ways that give rise to different preferences, contrary to the invariance criterion of rational choice. The process of mental accounting, in which people organize the outcomes of transactions, explains some anomalies of consumer behavior. In particular, the acceptability of an option can depend on whether a negative outcome is evaluated as a cost or as an uncompensated loss. The relationships between decision values and experience values and between hedonic experience and objective states are discussed.

Gigerenzer’s Critique

Is sometimes based on framing effects

Showing that there are less “mistakes” in a different framing

Well, all of mathematics is about framing…

Back to Framing Effects

Additional examples:

- Cash discount
- Tax deductions

What can be done about them?

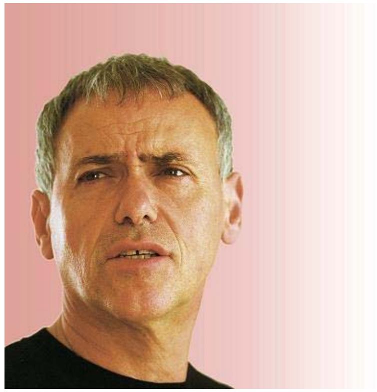
- The role of formal models

Framing Effects

That representations do not matter is not even a formal assumption

Implicitly assumed away in economic theory

- (It is not even an explicit assumption, apart from a few exceptions such as Rubinstein)



Ariel Rubinstein (b. 1951)

Betting?

A: You are given $\$1,000$ for sure. Which of the following two options would you prefer?

a. to get additional $\$500$ for sure;

b. to get another $\$1,000$ with probability 50%, and otherwise – nothing more (and be left with the first $\$1,000$).

B: You are given $\$2,000$ for sure. Which of the following two options would you prefer?

a. to lose $\$500$ for sure;

b. to lose $\$1,000$ with probability 50%, and otherwise – to lose nothing.

In both versions

the choice is between:

a. $\$1{,}500$ for sure;

b. $\$1{,}000$ with probability 50%,

and $\$2{,}000$ with probability 50%.

Loss Aversion,

Status Quo Bias,

Endowment Effect

Gain-Loss Asymmetry

Loss aversion

Relative to a reference point

Risk aversion in the domain of gains, but loss aversion in the domain of losses

Is it rational to fear losses?

Three scenarios:

- The politician
- The spouse
- The self

The same mode of behavior may be rational in some domains but not in others

Endowment Effect

What's the worth of a coffee mug?

- How much would you pay to buy it?
- What gift would be equivalent?
- How much would you demand to sell it?

Should all three be the same?

Standard economic analysis

Suppose that you have m dollars and 0 mugs

- How much would you pay to buy it?

$$(m - p, 1) \sim (m, 0)$$

- What gift would be equivalent?

$$(m + q, 0) \sim (m, 1)$$

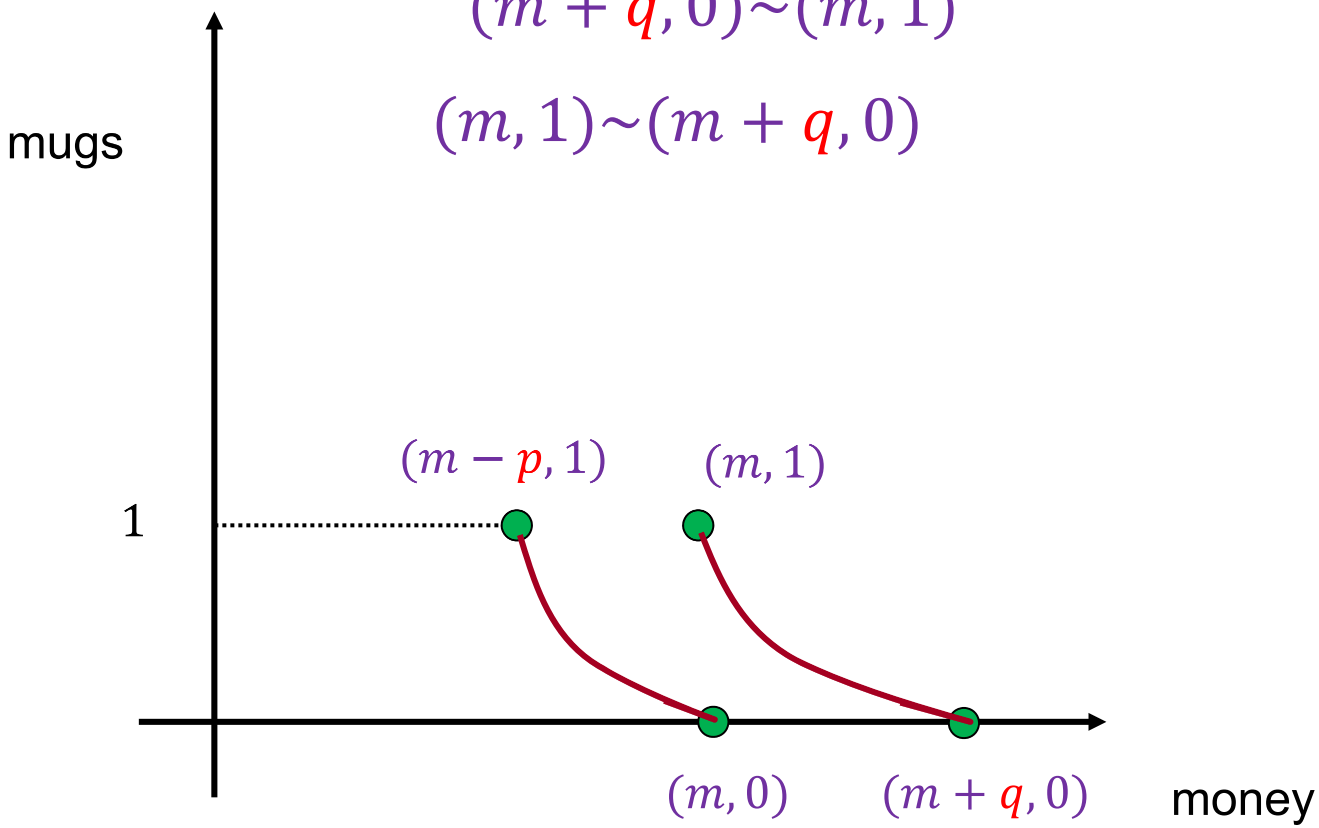
- How much would you demand to sell it?

$$(m, 1) \sim (m + q, 0)$$

Standard economic analysis

$$(m+q,0) \sim (m,1)$$

$$(m,1) \sim (m+q,0)$$



Standard analysis – conclusion

In short, of the three questions,

- How much would you pay to buy it?
- What gift would be equivalent?
- How much would you demand to sell it?

– (only) the last two should be the same

Results of mug experiment

- How much would you pay to buy it?

  $2.87

- What gift would be equivalent?

  $3.12

- How much would you demand to sell it?

  $7.12

The Endowment Effect

We tend to value what we have more than what we still don't have

A special case of "status quo bias"

Related to the "disposition effect"

Reference

Toward a positive theory of consumer choice

Richard H. Thaler

Journal of Economic Behavior and Organization Vol. 1 (1980) pp. 39-60.

Abstract

The economic theory of the consumer is a combination of positive and normative theories. Since it is based on a rational maximizing model it describes how consumers should choose, but it is alleged to also describe how they do choose. This paper argues that in certain well-defined situations many consumers act in a manner that is inconsistent with economic theory. In these situations economic theory will make systematic errors in predicting behavior. Kahneman and Tversky's prospect theory is proposed as the basis for an alternative descriptive theory. Topics discussed are: underweighting of opportunity costs, failure to ignore sunk costs, search behavior, choosing not to choose and regret, and precommitment and self-control.

Reference

Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias

Daniel Kahneman, Jack L. Knetsch, Richard H. Thaler

Journal of Economic Perspectives Vol. 5 No. 1 (Winter 1991) pp. 193-206

Abstract

A wine-loving economist we know purchased some nice Bordeaux wines years ago at low prices. The wines have greatly appreciated in value, so that a bottle that cost only $10 when purchased would now fetch $200 at auction. This economist now drinks some of this wine occasionally, but would neither be willing to sell the wine at the auction price nor buy an additional bottle at that price. Thaler (1980) called this pattern—the fact that people often demand much more to give up an object than they would be willing to pay to acquire it—the endowment effect. The example also illustrates what Samuelson and Zeckhauser (1988) call a status quo bias, a preference for the current state that biases the economist against both buying and selling his wine. These anomalies are a manifestation of an asymmetry of value that Kahneman and Tversky (1984) call loss aversion—the disutility of giving up an object is greater that the utility associated with acquiring it. This column documents the evidence supporting endowment effects and status quo biases, and discusses their relation to loss aversion.

Recent Survey

Explanations of the endowment effect: an integrative view

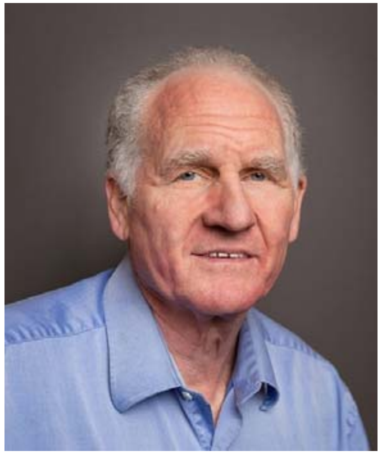
Carey K. Morewedge, Colleen E. Giblin

Trends in Cognitive Science Vol. 19 No. 6 (June 2015) pp. 339-348

Abstract

The endowment effect is the tendency for people who own a good to value it more than people who do not. Its economic impact is consequential. It creates market inefficiencies and irregularities in valuation such as differences between buyers and sellers, reluctance to trade, and mere ownership effects. Traditionally, the endowment effect has been attributed to loss aversion causing sellers of a good to value it more than buyers. New theories and findings – some inconsistent with loss aversion – suggest evolutionary, strategic, and more basic cognitive origins. In an integrative review, we propose that all three major instantiations of the endowment effect are attributable to exogenously and endogenously induced cognitive frames that bias which information is accessible during valuation.

The Status Quo Bias



William Samuelson (b. 1952)

Richard Zeckhauser (b. 1940)

"individuals disproportionately stick with the status quo"

Reference

Status quo bias in decision making

William Samuelson, Richard Zeckhauser

Journal of Risk and Uncertainty Vol. 1 (1988) pp. 7-59

Abstract

Most real decisions, unlike those of economics texts, have a status quo alternative—that is, doing nothing or maintaining one's current or previous decision. A series of decision-making experiments shows that individuals disproportionately stick with the status quo. Data on the selections of health plans and retirement programs by faculty members reveal that the status quo bias is substantial in important real decisions. Economics, psychology, and decision theory provide possible explanations for this bias. Applications are discussed ranging from marketing techniques, to industrial organization, to the advance of science.

The Disposition Effect

People tend to hold on to stocks that lost in value and to sell stocks that gained

Hersh Shefrin (b. 1948)

Meir Statman

Reference

The Disposition to Sell Winners too Early and Ride Losers too Long

Hersh Shefrin, Meir Statman

Journal of Finance Vol. 40 (1985) pp. 777-790

Abstract

One of the most significant and unique features in Kahneman and Tversky's approach to choice under uncertainty is aversion to loss realization. This paper is concerned with two aspects of this feature. First, we place this behavior pattern into a wider theoretical framework concerning a general disposition to sell winners too early and hold losers too long. This framework includes other elements, namely mental accounting, regret aversion, self-control, and tax considerations. Second, we discuss evidence which suggests that tax considerations alone cannot explain the observed patterns of loss and gain realization, and that the patterns are consistent with a combined effect of tax considerations and the three other elements of our framework. We also show that the concentration of loss realizations in December is not consistent with fully rational behavior, but is consistent with our theory.

Is it rational

… to value

a house
your grandfather's pen
your car
equity

…more just because it's yours?

Rationalization of Endowment Effect

- Information: there is less uncertainty about products we know
  - Someone else's used car
- Stabilization of choice
- Transaction costs
  - Getting used to a new computer system

Sunk Cost – example

- You go to a movie. It was supposed to be good, but it turns out to be boring. Would you leave in the middle and do something else instead?

- Your friend had a ticket to a movie. She couldn't make it, and gave you the ticket "instead of just throwing it away". The movie was supposed to be good, but it turns out to be boring. Would you leave in the middle and do something else instead?

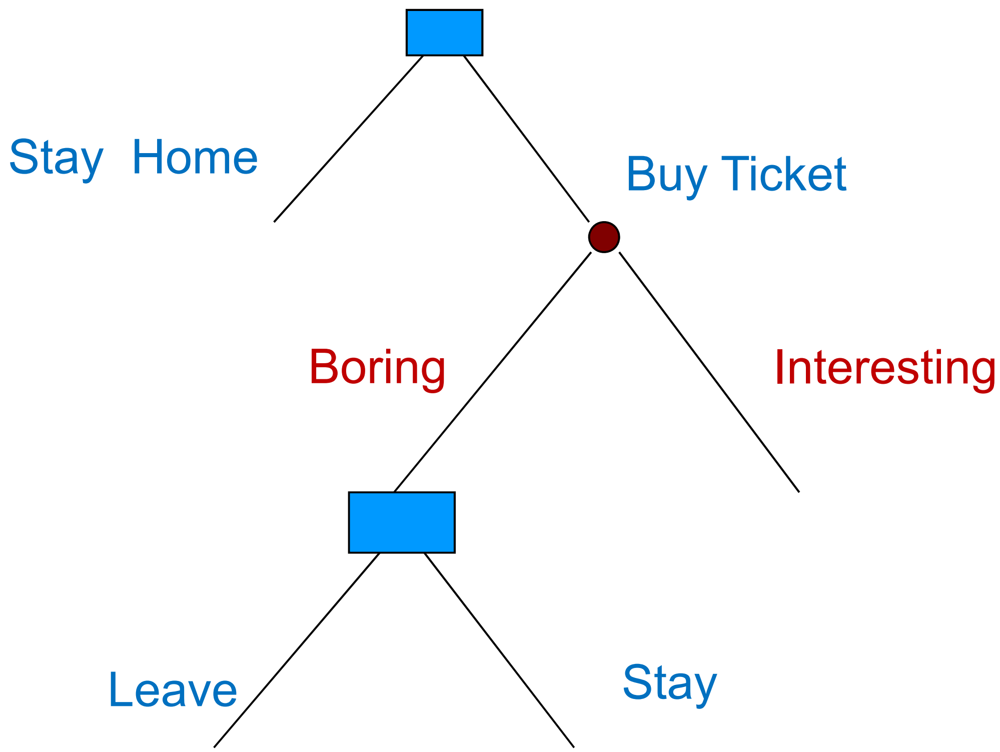
Sunk Cost

- Cost that is "sunk" should be ignored
- Often, it's not
- Additional examples:
  - Switching to another line (or another lane) when yours is evidently very slow
  - Eating more than you really want just because you already paid for the food

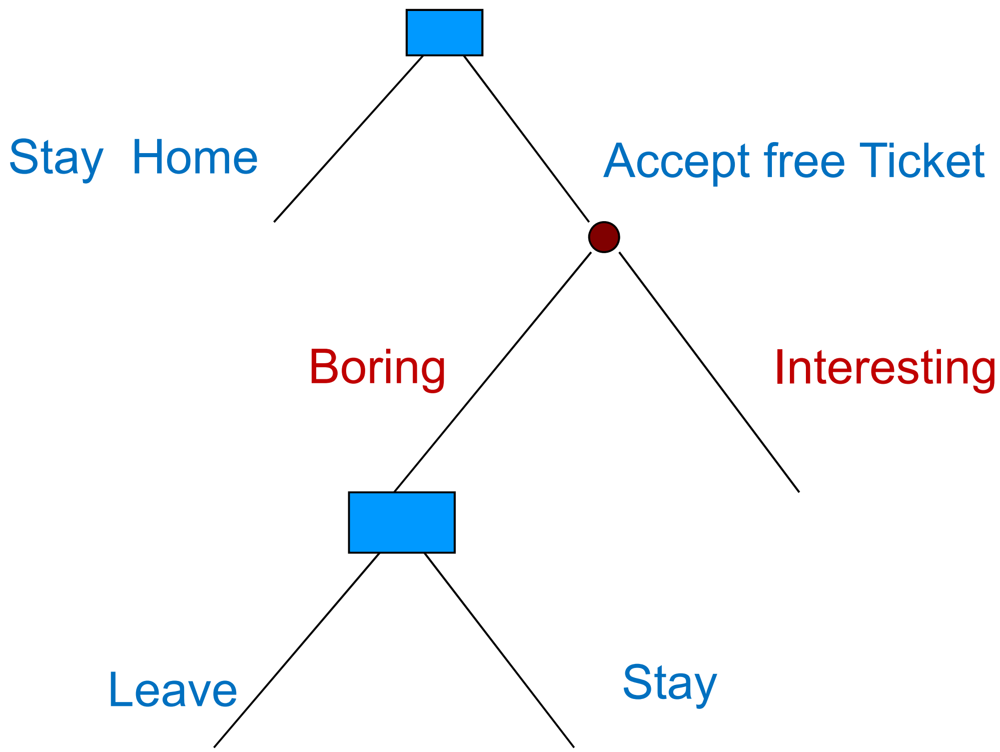
Is the Sunk Cost Effect rational?

- Well, it makes us feel bad to admit a mistake
- And it makes us look bad in the eyes of others
- But if we don’t like it, what can be done to avoid the sunk cost effect?

Decision Tree I



Decision Tree II



Consequentialism

- Only the consequences matter
- The decision at a node in the tree depends only on the subtree that starts there
- Not on how we got there and which other subtrees we could have been at
- Helps ignore sunk costs if we so wish

What's in a consequence?

- Does consequentialism mean we'll be ungrateful to our old teachers?
- Not necessarily: history can be part of the "consequence"
- If we push it too far, consequentialism would be vacuous. The formal model helps us decide how much of the history we wish to put into the notion of "a consequence"

Menu Effects

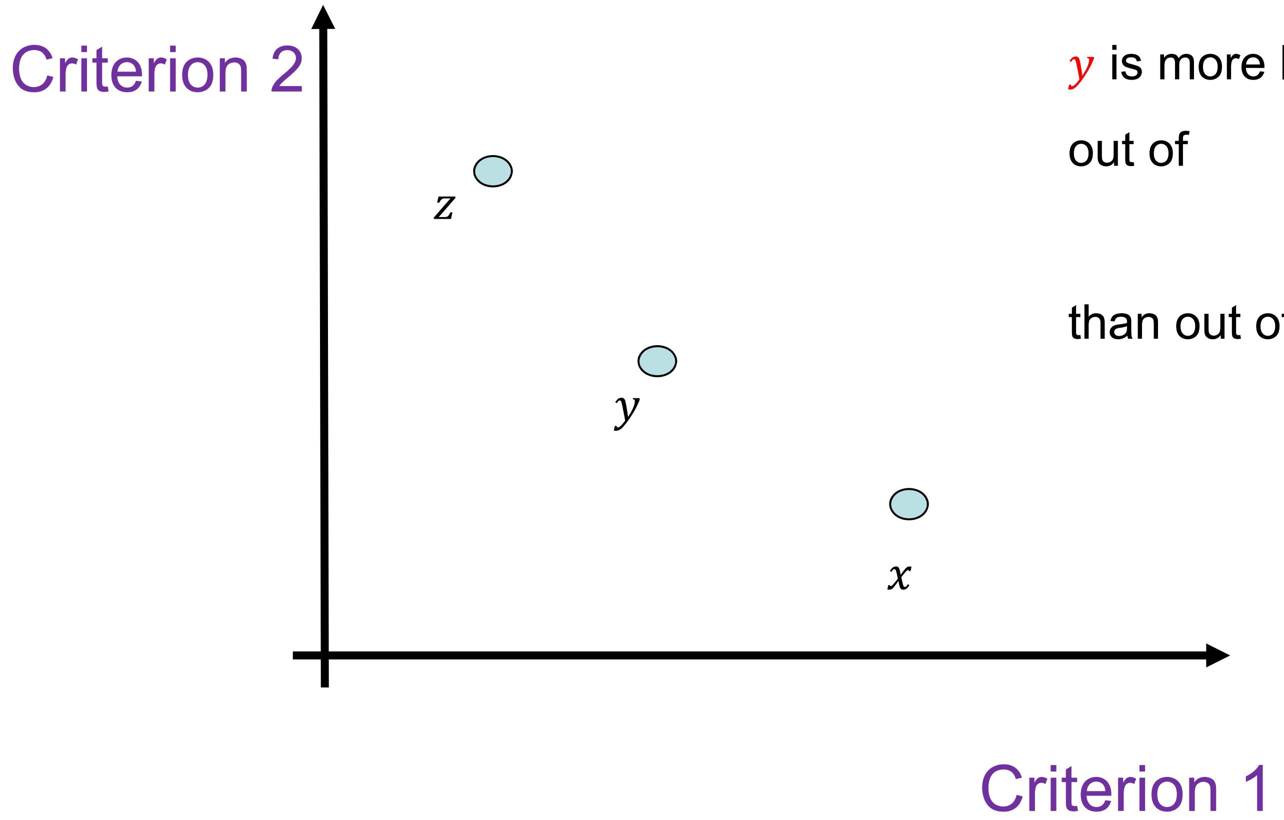
President Trump

From Jan 4th NY Times:

WASHINGTON — In the chaotic days leading to the death of Maj. Gen. Qassim Suleimani, Iran's most powerful commander, top American military officials put the option of killing him — which they viewed as the most extreme response to recent Iranian-led violence in Iraq — on the menu they presented to President Trump.

They didn't think he would take it. In the wars waged since the Sept. 11, 2001, attacks, Pentagon officials have often offered improbable options to presidents to make other possibilities appear more palatable.

The Compromise Effect



y is more likely to be chosen

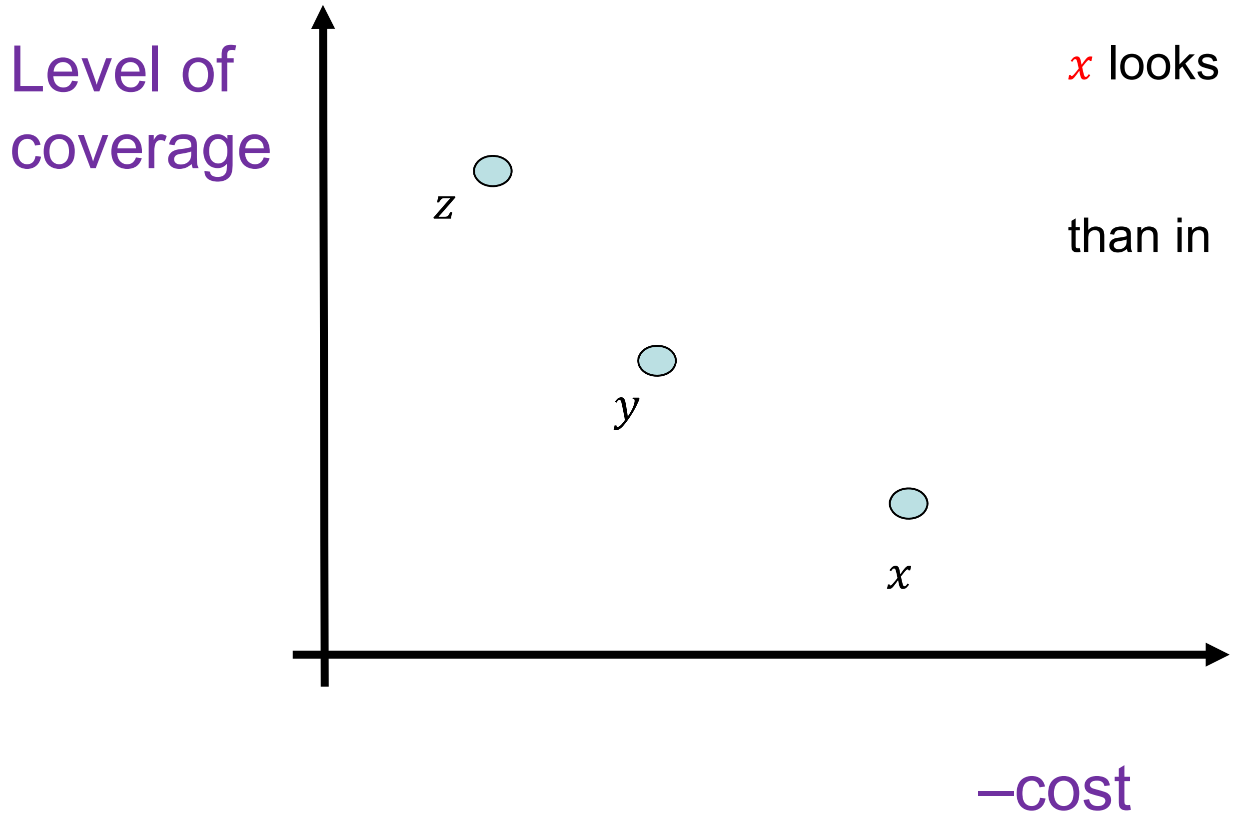
out of

$\{x, y, z\}$

than out of

$\{x, y\}$

Example: choice of health plan



x looks more extreme in

$\{x, y, z\}$

than in

$\{x, y\}$

Is the Compromise Effect rational?

- Could be a way to save cognitive resources
- Requires some implicit theory about who put options on the menu and why
- But taking these into account is actually implied by rationality

Was Trump rational?

- One story: sure; the consultants were trying to trick him via the compromise effect but he wouldn't budge

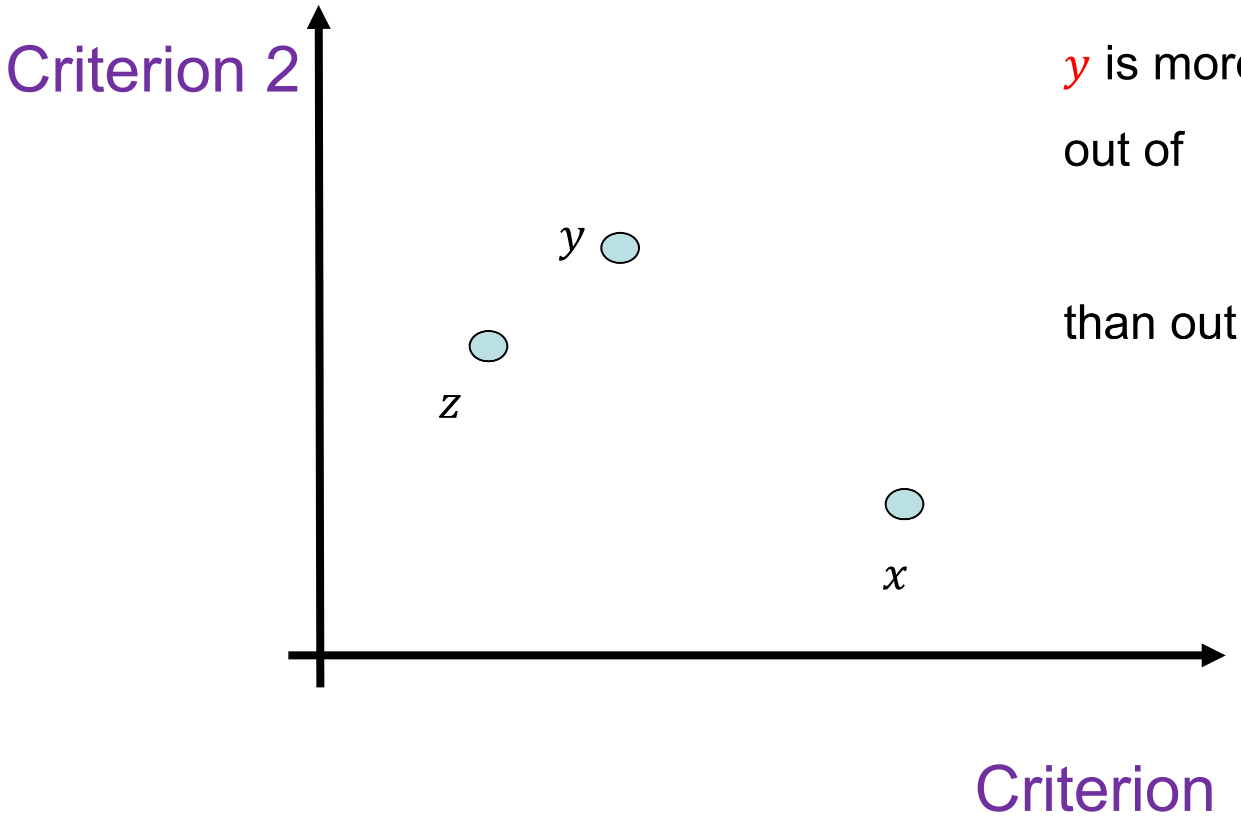
- Another story: no; he failed to understand that the consultants were giving him more information by the selection of the menu

The IIA

(Independence of Irrelevant Alternatives)

- If x appears to be preferred to y in the context of one menu, this shouldn't be reversed in another
  - ("appears to be preferred" – observed to be chosen where the other is also available)
- Violated when the Compromise Effect is in action

The Decoy Effect



y is more likely to be chosen

out of

$$\{x, y, z\}$$

than out of

$$\{x, y\}$$

Other violations of the IIA

- Order the second-least-expensive wine on the menu
- If the costs are $x > y > z$

select

x out of $\{x, y\}$

but

y out of $\{x, y, z\}$

Is that rational?

- Two types of rationalizations:
  - Subtle information effects
  - Psychological payoffs
- Behavioral economics contributed in
  - Awareness to some subtleties
  - Psychological determinants of the utility
- In any event, if we look at bare data, the IIA might be violated

Other menu effects

- Changing the default
- The 401K example
- Organ donation

Reference

The power of suggestion: Inertia in 401(K) participation and savings behavior

Brigitte C. Madrian, Dennis Shea

Quarterly Journal of Economics Vol. 116 No. 4 (Nov. 2011) pp. 1149-1187

Abstract

This paper analyzes the impact of automatic enrollment on 401(k) savings behavior. We have two key findings. First, 401(k) participation is significantly higher under automatic enrollment. Second, a substantial fraction of 401(k) participants hired under automatic enrollment retain both the default contribution rate and fund allocation even though few employees hired before automatic enrollment picked this particular outcome. This "default" behavior appears to result from participant inertia and from employee perceptions of the default as investment advice. These findings have implications for the design of 401(k) savings plans as well as for any type of Social Security reform that includes personal accounts over which individuals have control. They also shed light more generally on the importance of both economic and noneconomic (behavioral) factors in the determination of individual savings behavior.

Is the Default Effect rational?

Well:

- The default tells me something about the options
- I might also not want to be among the few who selected differently from most

Again, two types of rationalizations:

- Subtle information effects
- Psychological payoffs

Assessing Likelihood

Linda

Linda is 31 years old… etc.

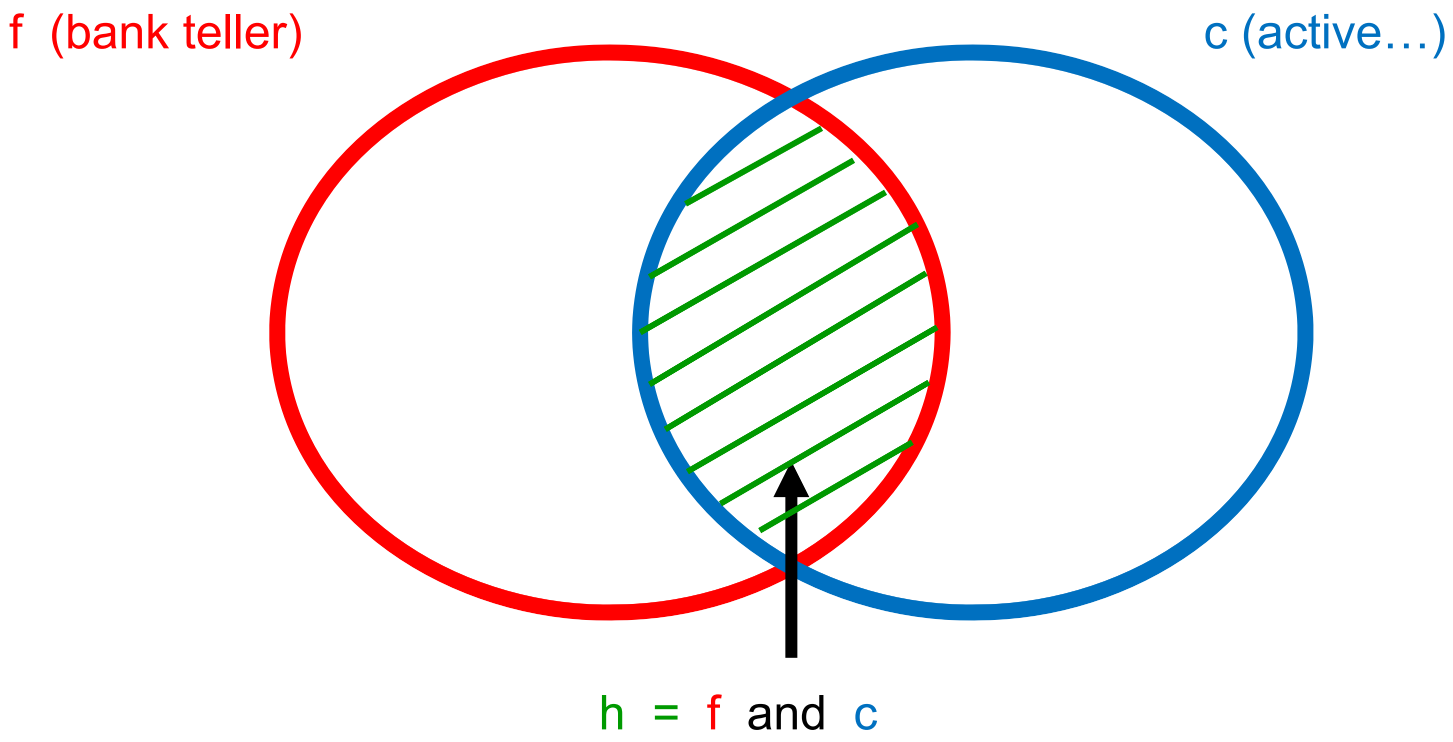
Did you rank

f. Linda is a bank teller

below

h. Linda is a bank teller who is active in a feminist movement ?

The Conjunction Fallacy



A conjunction can't be more likely than any of the conjuncts!

What's behind the conjunction fallacy?

Many explanations:

– "a bank teller" – "a bank teller who is not active"?

– Ranking propositions is not a very natural task

In particular, it may be the case that people implicitly switch to the question

"Is this really the same Linda?"

rather than

"Is this proposition true?"

Is this person telling the truth?

As with a witness in court, more details, provided they're coherent, increase credibility.

How come?

The more the witness tells us, the less likely is the conjunction of her statements

But we're not asking

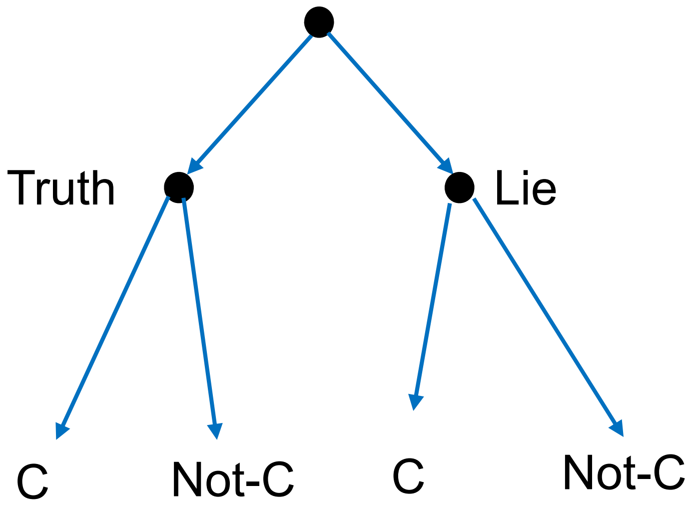
> "How likely is this conjunction of propositions?"

but

> "Is this witness telling us the truth?"

Is the witness telling the truth?

Bayesian inference



C – consistent testimony

What’s behind the conjunction fallacy?

Kahneman and Tversky:

– There is a Representativeness Heuristic at work, and it can be misleading

– Being a bank teller doesn’t seem representative of Linda

– A bank teller who’s active in a feminist movement – more like the Linda we know

What’s a “heuristic” ?

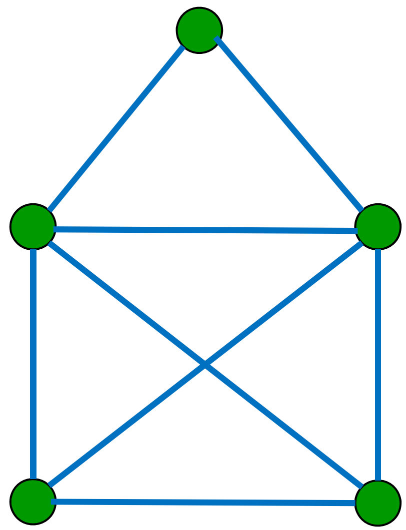
A method that helps us deal with a complex problem, suggesting a solution that typically makes sense but isn’t guaranteed to be optimal

The term is used both in CS (computer science)/ OR(operation research) and in psychology

Allow me a medium-length digression…

Example of an algorithm

Problem: Can you draw a path that goes through each edge exactly once?



That was an Euler path

A (connected, undirected) graph has an Euler path if and only if, when we count the number of edges that go through each node (the node's "degree"), we find either all even numbers, or all even but two (that are odd).

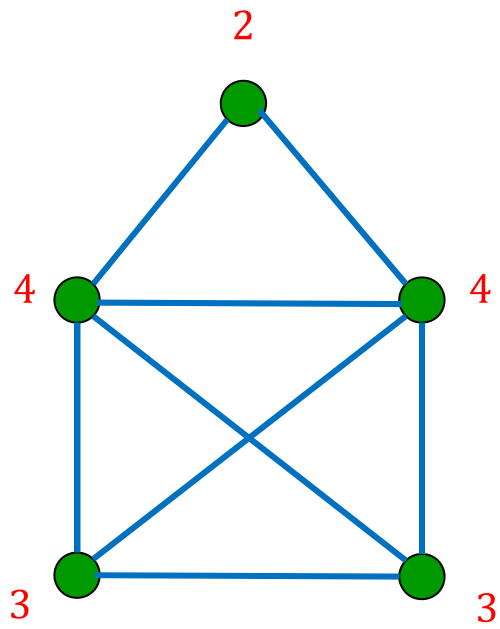
Leonhard Euler (1707-1783)

(What about "all but one"?)

The “degrees” in the example

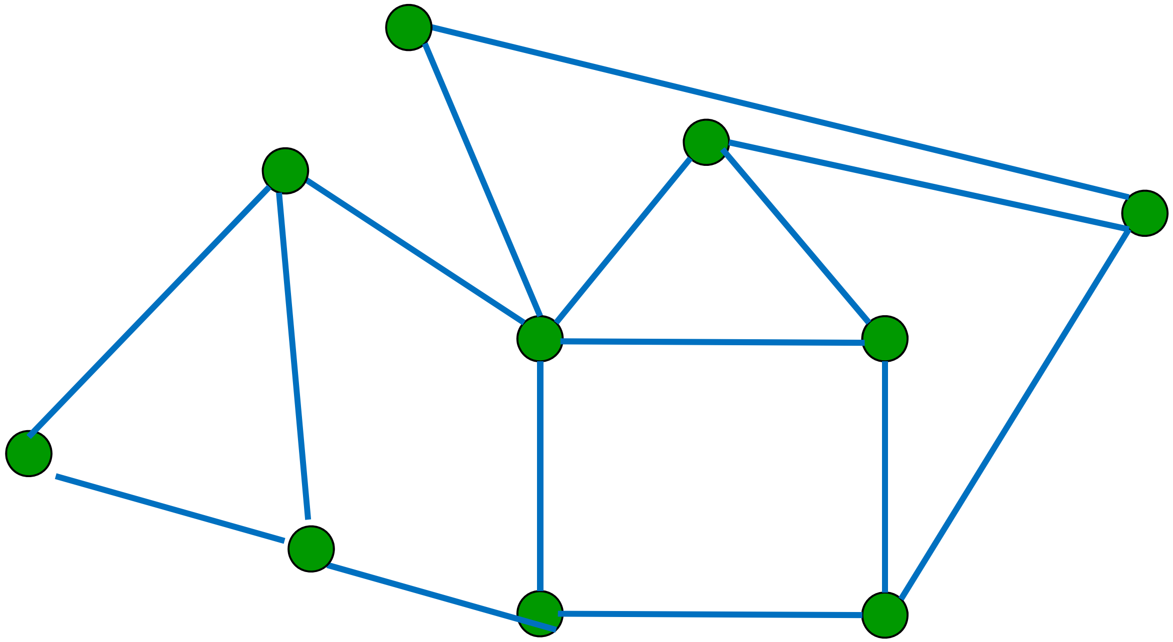
The counting shows that an Euler path exists here

(and the counting will also give us a hint as to where to start):



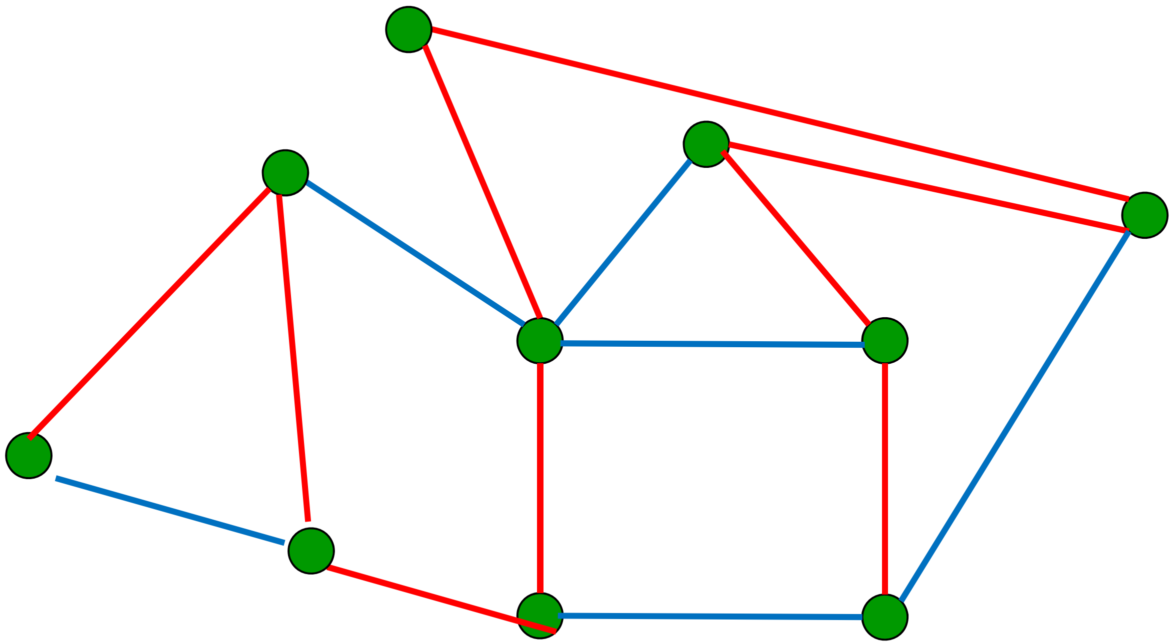
And how about…

Problem: Can you draw a path that goes through each node exactly once (a Hamiltonian path) ?



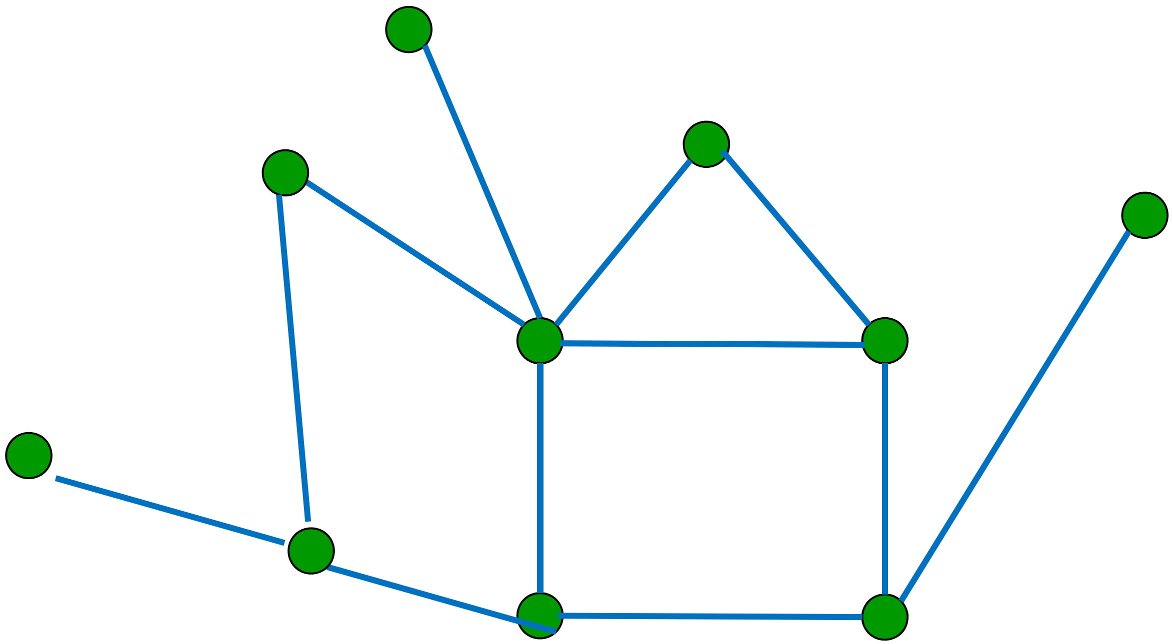
For instance

A Hamiltonian path exists in this example:



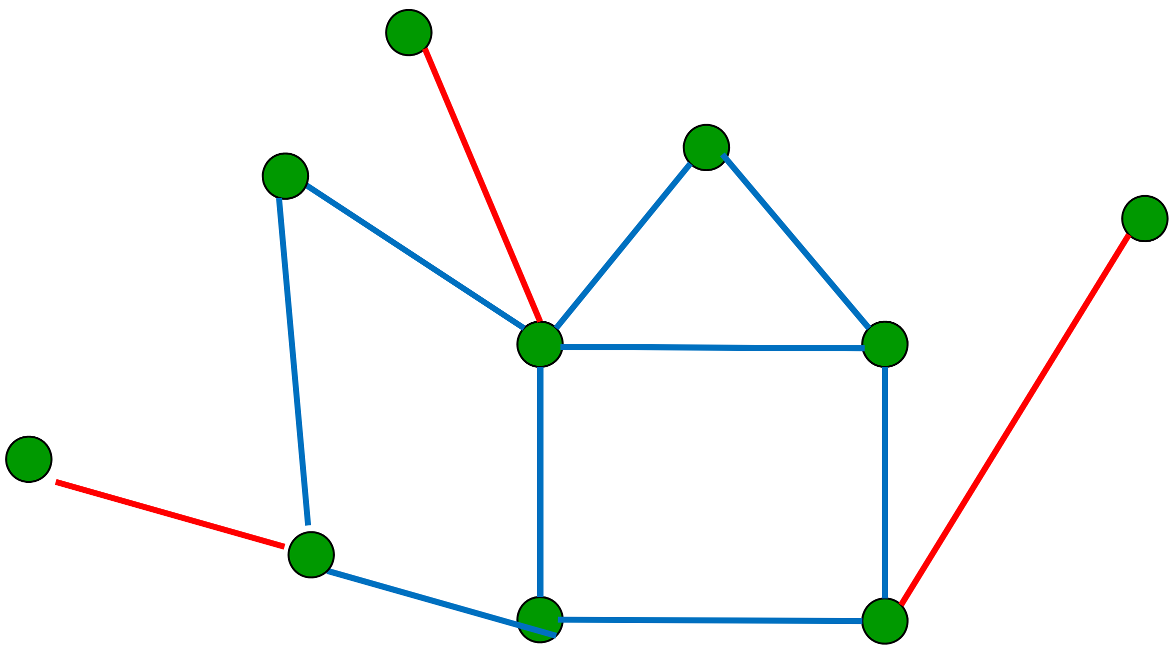
But

A Hamiltonian path doesn’t exists in :



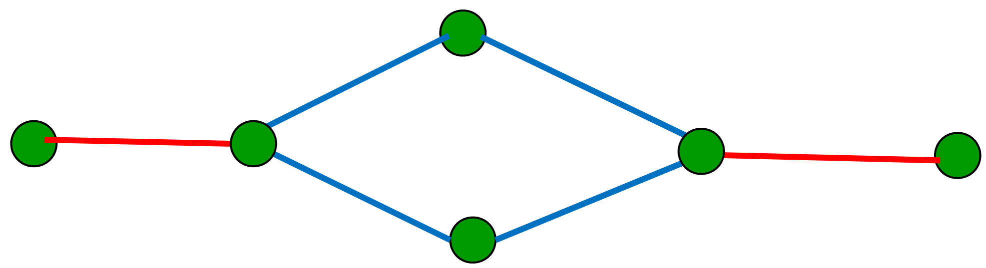
Because…

This graph has too many (3) nodes of degree 1:



Can we just count nodes of degree 1?

No:



This graph has only two such nodes, yet no Hamiltonian path

Instead of being so clever

Let’s just try all possibilities

If a Hamiltonian path exists, it is simply a permutation (ordering) of the nodes such that any two consecutive nodes are connected by an edge

Let’s try them all!

Brute force

"Try all possible permutations" –

Easier said than done…

The number of atoms in the universe is estimated to be in the range

$10^{78} - 10^{82}$

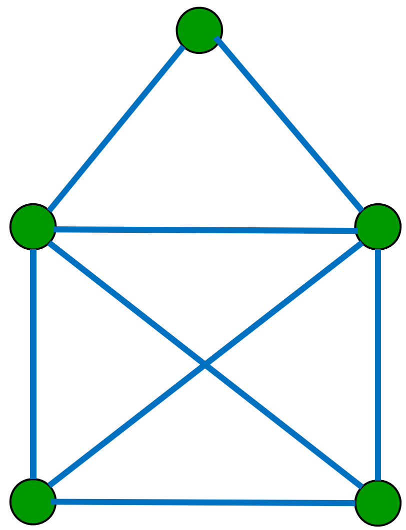
So even parallel computing is hopeless

n	$n!$
1	1
2	2
3	6
4	24
10	$3{,}628{,}800$
30	$2.65 * 10^{32}$
60	$8.32 * 10^{81}$

Wait, but…

With an Euler path, we could also try all permutations of the edges – in our example $8! = 40{,}320$ – but we found something much smarter!

(OK, Euler did.)



What is “much smarter”?

Polynomial complexity of an algorithm – solves the problem for n data points, at the worst case, in a number of steps that is no more than a polynomial in n

$$n\ , n^2, n^5, \ldots$$

Exponential complexity of an algorithm – might take a number of steps that grows exponentially in n

$$2^n, n!, \ldots$$

Polynomial problems

A problem is polynomial if there exists at least one algorithm that can solve it in polynomial time complexity

Maybe the Hamiltonian Path problem is polynomial?

Maybe. We don't know.

Classes of (yes/no) problems

P (Polynomial) – A solution can be found in polynomial time

NP (Nondeterministic Polynomial) – A suggested solution can be verified in polynomial time

"Does there exists a Hamiltonian path?" is in NP

(A bit like the distinction between recall and recognition in psychology)

NP-Completeness

There is a problem about which the following is true:

IF you could solve it in polynomial time, THEN you can solve any problem in NP in polynomial time

(This isn't the definition of NP-Completeness)

Results from 1971, 1973

Stephen Cook (b. 1939)

Leonid Levin (b. 1948)

Many problems are NP-Complete

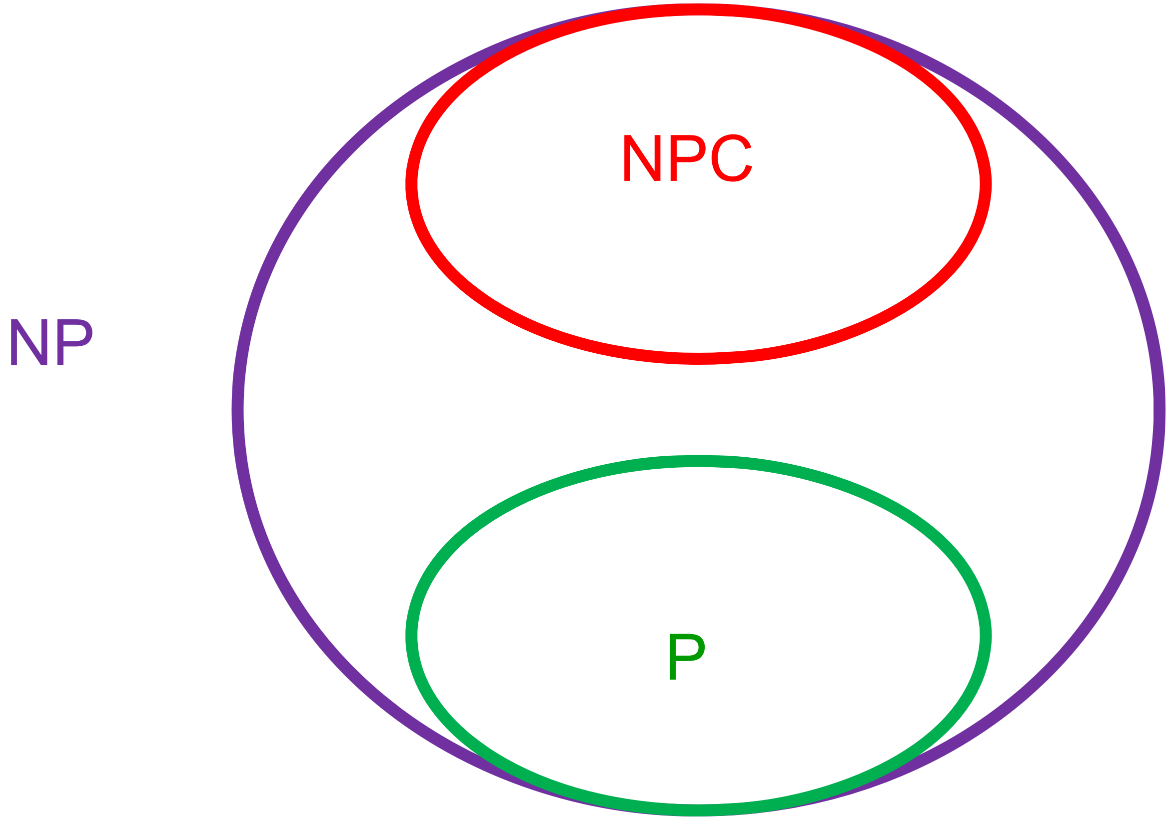
1972 Karp showed that 21 problems are NP-Complete

1979 Garey and Johnson publish a book with many more

You can find a catalog at https://www.nada.kth.se/~viggo/problemlist/compendium.html

Richard Karp (b. 1935)

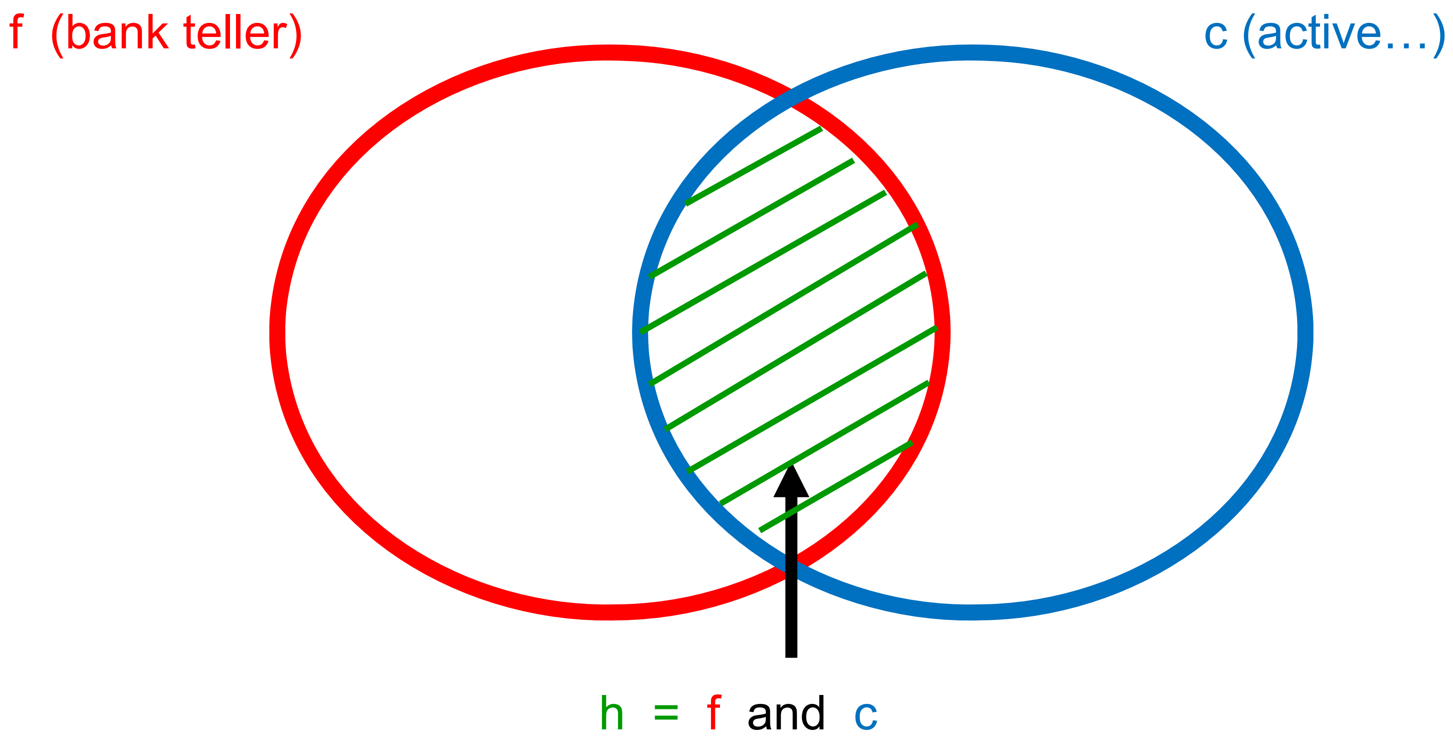
P = NP ?



Why am I telling you all this?

- Computer scientists also use heuristics
- Some problems that economic theory assumes people solve are NP-Complete
- And that brings about questions about “rationality”
- For now: if the best computer scientists sometimes can only offer heuristics, it makes sense that so does the human brain

Back to Linda



One way to be immune to the Conjunction Fallacy is to use

subjective probabilities

Subjective probabilities

- Formal models immune against framing effects
- Subjective probabilities – against the Conjunction Fallacy and related mistakes
- In both cases, the model won't provide the answer
- But it will help you avoid certain patterns that you may consider mistakes

(Truth is, other models can also immune against the Conjunction Fallacy)

Words in a novel

A: In four pages of a novel (about 2,000 words) in English, do you expect to find more than ten words that have the form _ _ _ _ _ *n* _ (seven-letter words that have the letter *n* in the sixth position)?

B: In four pages of a novel (about 2,000 words) in English, do you expect to find more than ten words that have the form _ _ _ _ *ing* (seven-letter words that end with *ing*)?

Availability heuristic

In the absence of a “scientific” database, we use our memory

Typically, a great idea

Sometimes, results in a biased sample

Probability of a specific cause

A: What is the probability that, in the next 2 years, there will be a cure for AIDS?

B: What is the probability that, in the next 2 years, there will be a new genetic discovery in the study of apes, and a cure for AIDS?

Availability heuristic

Unpacking Effect

A: What is the probability that, during the next year, your car would be a "total loss" due to an accident?

B: What is the probability that, during the next year, your car would be a "total loss" due to:

a. an accident in which the other driver is drunk?
b. an accident for which you are responsible?
c. an accident occurring while your car is parked on the street?
d. an accident occurring while your car is parked in a garage?
e. one of the above?

Availability heuristic

Reference

The Unpacking Effect in evaluative judgments: when the whole is less than the sum of its parts

Nicholas Epley, Leaf Van Boven

Journal of Experimental Social Psychology, Vol. 39 (2003), pp. 263-269

Abstract

Any category or event can be described in more or less detail. Although these different descriptions can reflect the same event objectively, they may not reflect the same event subjectively. Research on Support Theory led us to predict that more detailed descriptions would produce more extreme evaluations of categories or events than less detailed descriptions. Four experiments demonstrated this unpacking effect when people were presented with (Experiments 1 and 4), generated (Experiment 2), or were primed with (Experiment 3) more rather than less detailed descriptions of events. This effect was diminished when the details were less personally relevant (Experiment 4). We discuss several psychological mechanisms, moderators, and extensions of the unpacking effect.

Reporting bias

Which of the following causes more deaths each year:

a. Digestive diseases
b. Motor vehicle accidents?

In the original KT study

(In the US 1981-1984)

Which of the following causes more deaths each year:

a. Stomach cancer
b. Motor vehicle accidents (MVA)?

The also collected data on media stories. For every story on stomach cancer death there were 147 on MVA death

Salary estimate

A newly hired engineer for a computer firm in Melbourne has four years of experience and good all-around qualifications.

Do you think that her annual salary is above or below [A: $65,000; B: $135,000] ? ______

What is your estimate?

Anchoring heuristic

In the absence of solid data, any number can be used as an “anchor”

Is it rational?

In K-T’s original formulation, someone “who knows little” said something

But there’s still some information in that

I only asked you what you thought about a given value

And yet, there’s some information in that, too

Anchoring heuristic

Can be used strategically

Should an employee be promoted / retained / fired ?

By talking about one option first you can affect the outcome

Especially if most people don’t really have an opinion

Mental Accounting

Losing a ticket

A: You have a ticket to a concert, which cost you $50. When you arrive at the concert hall, you find out that you lost the ticket. Would you buy another one (assuming you have enough money in your wallet)?

B: You are going to a concert. Tickets cost $50. When you arrive at the concert hall, you find out that you lost a $50 bill. Would you still buy the ticket (assuming you have enough money in your wallet)?

Mental Accounting

- Different expenses come from “different” accounts
- People and households run “accounts” in their heads as if they were large organizations with budgets

Richard Thaler (b. 1945)

Mental Accounting examples

- Your spouse buys you the gift you didn't afford
  - Have you ever bought yourself a B-day present?
- You spend more on special occasions
  - Vacations

    OK, this may be due to "producing" the perfect vacation
  - Moving
- Spending money on a car's accessories

Reference

Mental Accounting and Consumer Choice

Richard Thaler

Marketing Science, Vol. 4 No. 3 (1985), pp. 199-214

Abstract

A new model of consumer behavior is developed using a hybrid of cognitive psychology and microeconomics. The development of the model starts with the mental coding of combinations of gains and losses using the prospect theory value function. Then the evaluation of purchases is modeled using the new concept of "transaction utility". The household budgeting process is also incorporated to complete the characterization of mental accounting. Several implications to marketing, particularly in the area of pricing, are developed.

Is Mental Accounting rational?

- The consumer problem is complex
- There are many choices

https://www.ted.com/talks/barry_schwartz_the_paradox_of_choice

- In fact, if there are indivisible goods, the consumer problem can be NP-Complete

("can"??? – well, it depends on the modeling)

Other rationalizations

- Helps cope with self-control problems

  How will I guarantee that I don't buy myself the sweater "only this time" every week?

- Uses external events as memory aids

  How many times this month have I bought the more expensive wine?

A possible definition

When we split the budget to sub-budgets recursively, we construct a DAG (Directed Acyclic Graph)

If it is a tree, all's fine

If not, we can assign a given expense to more than one possible budget

Dynamic Inconsistency

Choice problems

A: Which of the following two options do you prefer?

- Receiving $10 today
- Receiving $12 a week from today

B: Which of the following two options do you prefer?

- Receiving $10 50 weeks from today
- Receiving $12 51 weeks from today

The classical model

$$U(c_0, c_1, c_2, \dots)$$

$$= u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ \dots$$

$0 < \ \delta < 1$ – a discount factor

Paul A. Samuelson (1915-2009)

Three assumptions

$$U(c_0, c_1, c_2, \dots) = u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ \dots$$

We could have:

$$U(c_0, c_1, c_2, \dots) = u_0(c_0) + u_1(c_1) + u_2(c_2) + \cdots$$

$$= a_0 u(c_0) + a_1 u(c_1) + a_2 u(c_2) + \cdots$$

$$= u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ \dots$$

First assumption: additivity

$$U(c_0, c_1, c_2, \dots) = u_0(c_0) + u_1(c_1) + u_2(c_2) + \cdots$$

Do you prefer to stay at

$***$ hotels in year 1

$****$ hotels in year 2

$*****$ hotels in year 3

Or the other way around?

Does utility in a given period depend on consumption in past ones?

Second assumption: “same” utility

$$U(c_0, c_1, c_2, \dots) = u_0(c_0) + u_1(c_1) + u_2(c_2) + \cdots$$

$$= a_0 u(c_0) + a_1 u(c_1) + a_2 u(c_2) + \cdots$$

With positive coefficients

$$a_0, a_1, a_2, \dots > 0$$

Will my preference between ski vacations and concerts remain unchanged throughout my lifetime?

Third assumption: Dynamic Consistency

$$a_0 u(c_0) + a_1 u(c_1) + a_2 u(c_2) + \cdots$$

$$= u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ \ldots$$

The preference questions

$\$10$ today or $\$12$ a week from today

$$U(10,0,0,0,\ldots)\ ?\ U(0,12,0,0,\ldots)$$

$\$10$ 50 weeks from today or $\$12$ 51 weeks from today

$$U(0,0,\ldots,0,10,0,0,0,\ldots)\ ?\ U(0,0,\ldots 0,0,12,0,0,\ldots)$$

The difficulty

The classical model

$$U(c_0, c_1, c_2 \ \dots) = u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ \dots$$

with a discount factor $0 < \delta < 1$

Can’t explain

$$U(10,0,0,0, \dots) > U(0,12,0,0, \dots)$$

As well as

$$U(0,0, \dots, 0,10,0,0,0, \dots) < U(0,0, \dots 0,0,12,0,0, \dots)$$

In more detail

Assume $u(0) = 0$

$$U(10,0,0,0,\ldots) > U(0,12,0,0,\ldots)$$

means

$$u(10) > \delta u(12)$$

But

$$U(0,0,\ldots,0,10,0,0,0,\ldots) < U(0,0,\ldots 0,0,12,0,0,\ldots)$$

is equivalent to

$$\delta^{50}u(10) < \delta^{51}u(12)$$

Dynamic Consistency

- What we plan today to do tomorrow is indeed what we will choose to do tomorrow

- Violated in this example

- Other violations:
  - Tomorrow I'll start studying for the exam
  - Next week I'll quit smoking
  - Next year I'll start saving for retirement

By contrast

The classical model

$$U(c_0, c_1, c_2, \dots)$$

$$= u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ \dots$$

More or less follows from dynamic consistency

Tjalling C. Koopmans (1910-1985)

Dynamic Inconsistency or Impatience?

The marshmallow experiment

https://www.youtube.com/watch?v=Yo4WF3cSd9Q

https://www.youtube.com/watch?v=QX_oy9614HQ

It might be hard to tell whether the impatient kids are (also?) dynamically inconsistent

Hyperbolic Discounting

In

$$U(c_0, c_1, c_2, \dots) = a_0 u(c_0) + a_1 u(c_1) + a_2 u(c_2)$$

replace

$$a_t = \delta^t$$

by

$$a_t = \frac{1}{1 + kt}$$

The β – δ Model

David Laibson
(b. 1966)

Instead of

$$U(c_0, c_1, c_2, \dots) = a_0 u(c_0) + a_1 u(c_1) + a_2 u(c_2)$$

Give the first period an extra (relative) weight:

$$U(c_0, c_1, c_2 \ \dots) = u(c_0) + \beta[\delta u(c_1) + \delta^2 u(c_2) + \ \dots]$$

with a discount factor $0 < \beta, \delta < 1$

Reference

Golden Eggs and Hyperbolic Discounting

David Laibson

The Quarterly Journal of Economics, Vol. 112 No. 2 (May 1997), pp. 443-478

Abstract

Hyperbolic discount functions induce dynamically inconsistent preferences, implying a motive for consumers to constrain their own future choices. This paper analyzes the decisions of a hyperbolic consumer who has access to an imperfect commitment technology: an illiquid asset whose sale must be initiated one period before the sale proceeds are received. The model predicts that consumption tracks income, and the model explains why consumers have asset-specific marginal propensities to consume. The model suggests that financial innovation may have caused the ongoing decline in U. S. savings rates, since financial innovation increases liquidity, eliminating commitment opportunities. Finally, the model implies that financial market innovation may reduce welfare by providing “too much” liquidity.

Critique of the $\beta - \delta$ Model

Present-bias, quasi-hyperbolic discounting, and fixed costs

Jess Benhabib, Alberto Bisin, Andrew Schotter

Games and Economic Behavior, Vol. 69 No. 2 (July 2010), pp. 205-223

Abstract

In this paper we elicit preferences for money–time pairs via experimental techniques. We estimate a general specification of discounting that nests exponential and hyperbolic discounting, as well as various forms of *present bias*, including quasi-hyperbolic discounting.

We find that discount rates are high and decline with both delay and amount, as most of the previous literature. We also find clear evidence for present bias. When identifying the form of the present bias, little evidence for quasi-hyperbolic discounting is found. The data strongly favor instead a specification with a small present bias in the form of a fixed cost, of the order of $4 on average across subjects. With such a fixed cost the curvature of discounting is imprecisely estimated and both exponential and hyperbolic discounting cannot be rejected for several subjects.

Attitudes to dynamic inconsistency

Ted O'Donoghue

Matthew Rabin (b, 1963)

Donoghue and Rabin suggested to distinguish between naïve and sophisticated decision makers

Reference

Doing it now or later?

Ted O'Donoghue, Matthew Rabin

American Economic Review, Vol. 89 No. 1 (1999), pp. 103-124

Abstract

The authors examine self-control problems--modeled as time-inconsistent, present-biased preferences--in a model where a person must do an activity exactly once. They emphasize two distinctions: do activities involve immediate costs or immediate rewards, and are people sophisticated or naive about future self-control problems? Naive people procrastinate immediate-cost activities and preproperate--do too soon--immediate-reward activities. Sophistication mitigates procrastination but exacerbates preproperation. Moreover, with immediate costs, a small present bias can severely harm only naive people, whereas with immediate rewards it can severely harm only sophisticated people. Lessons for savings, addiction, and elsewhere are discussed.

In Summation

We promised to ask about each violation whether it is

- Robust?
  - Stable across experiments
  - Has external validity
- Relevant?
  - Occurs in economically relevant problems
  - Has ecological validity (for economics)
- Rational?
  - Cannot, or will not be easily fixed

Examples of my subjective impressions

	Robust	Relevant	Rational
The Conjunction Fallacy	–	–	–
Availability Bias	✓	✓	✓
Dynamic Inconsistency	✓	✓	–
Mental Accounting	✓	✓	✓

You should have your own

… But try to think in these terms

CONSUMING STATISTICAL DATA

Conditional Probabilities

Problem

A newly developed test for a rare disease has the following features: if you do not suffer from the disease, the probability that you test positive ("false positive") is 5%. However, if you do have the disease, the probability that the test fails to show ("false negative") is 10%.

You took the test, and, unfortunately, you tested positive. The probability that you have the disease is:

The missing piece

The a-priori probability of the disease,

$$P\,(D)\ =\ p$$

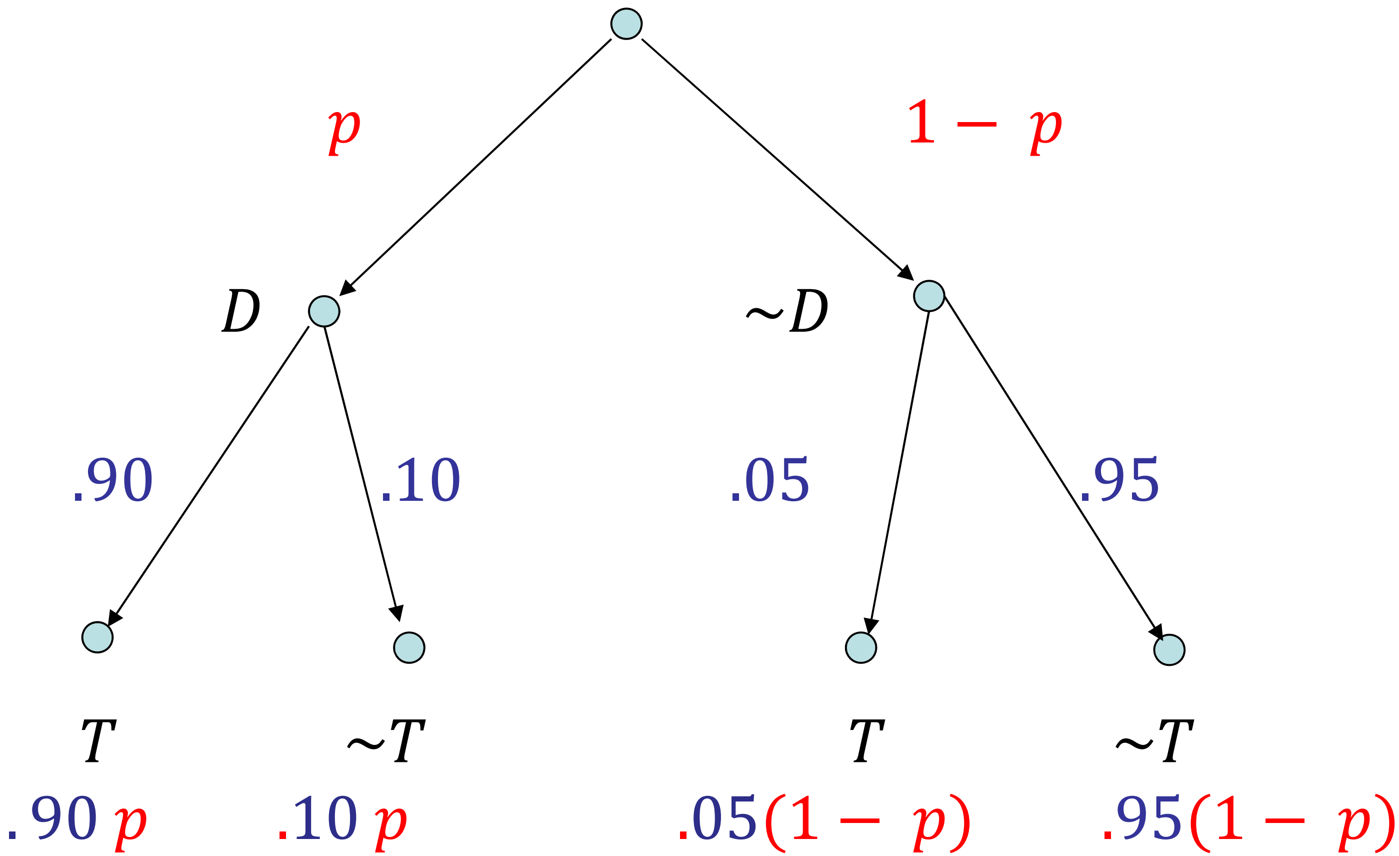
Intuitively, assume that $p = 0$ vs. $p = 1$

Maybe the disease is known to be extinct ($p = 0$)

- The accuracy of the test is unchanged, there are still false positives

Maybe I'm anyway diagnosed ($p = 1$)

Conditional probabilities



The calculation

$$P(D|T) = \frac{\mathrm{P}(D \cap T)}{\mathrm{P}(T)}$$

$$= \frac{.90p}{.90p + .05(1-p)}$$

with $p = P(D)$

… can indeed be anywhere between 0 and 1 !

For example…

If, say, $P(D) = .01$,

$$P(D|T) = \frac{.90p}{.90p + .05(1-p)}$$

$$= \frac{.01 * .90}{.01 * .90 + .99 * .05} = 15.3\%$$

So

$$P(D|T) > P(D)$$

but

$$P(D|T) < 50\%$$

That is,

$$P(D \mid T) > P(D)$$

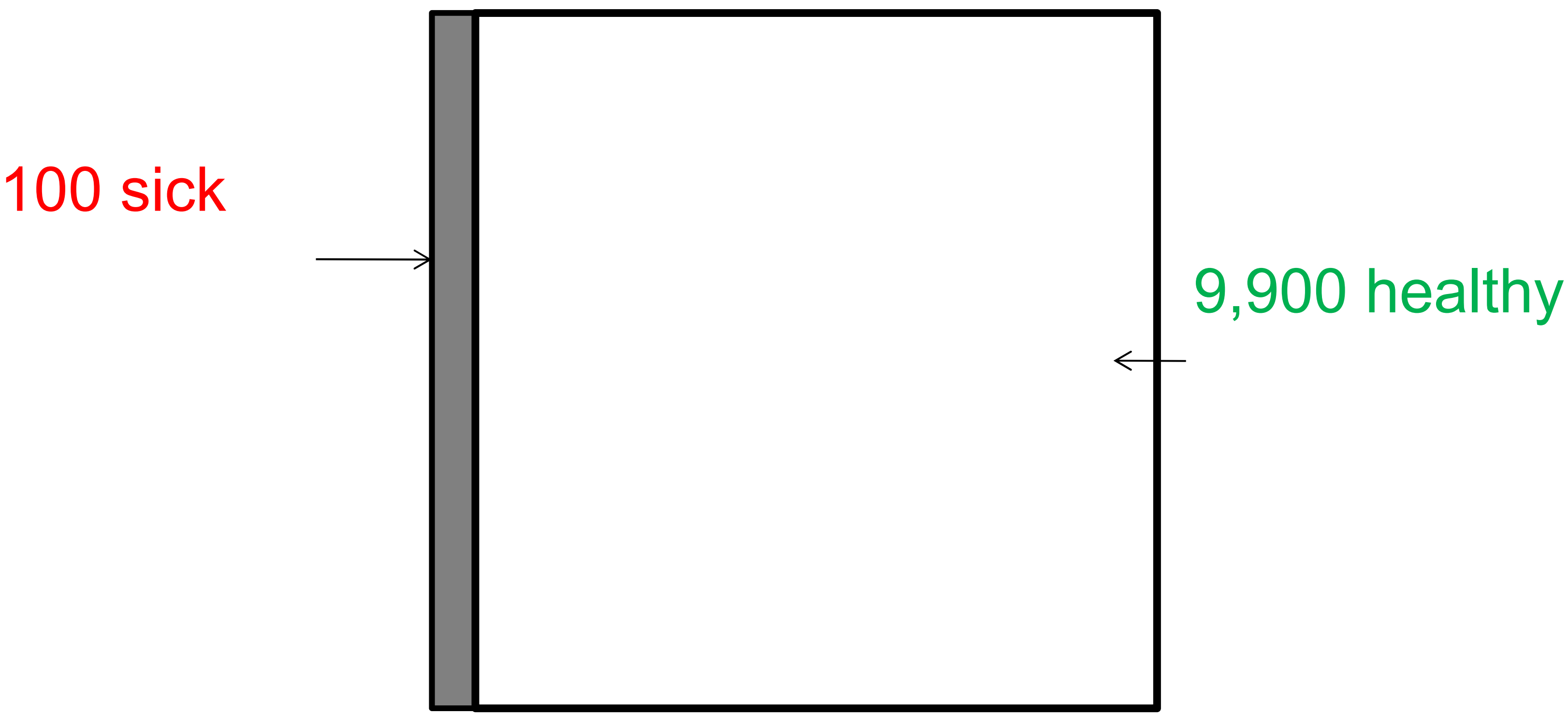
means that testing positive isn't good news

but

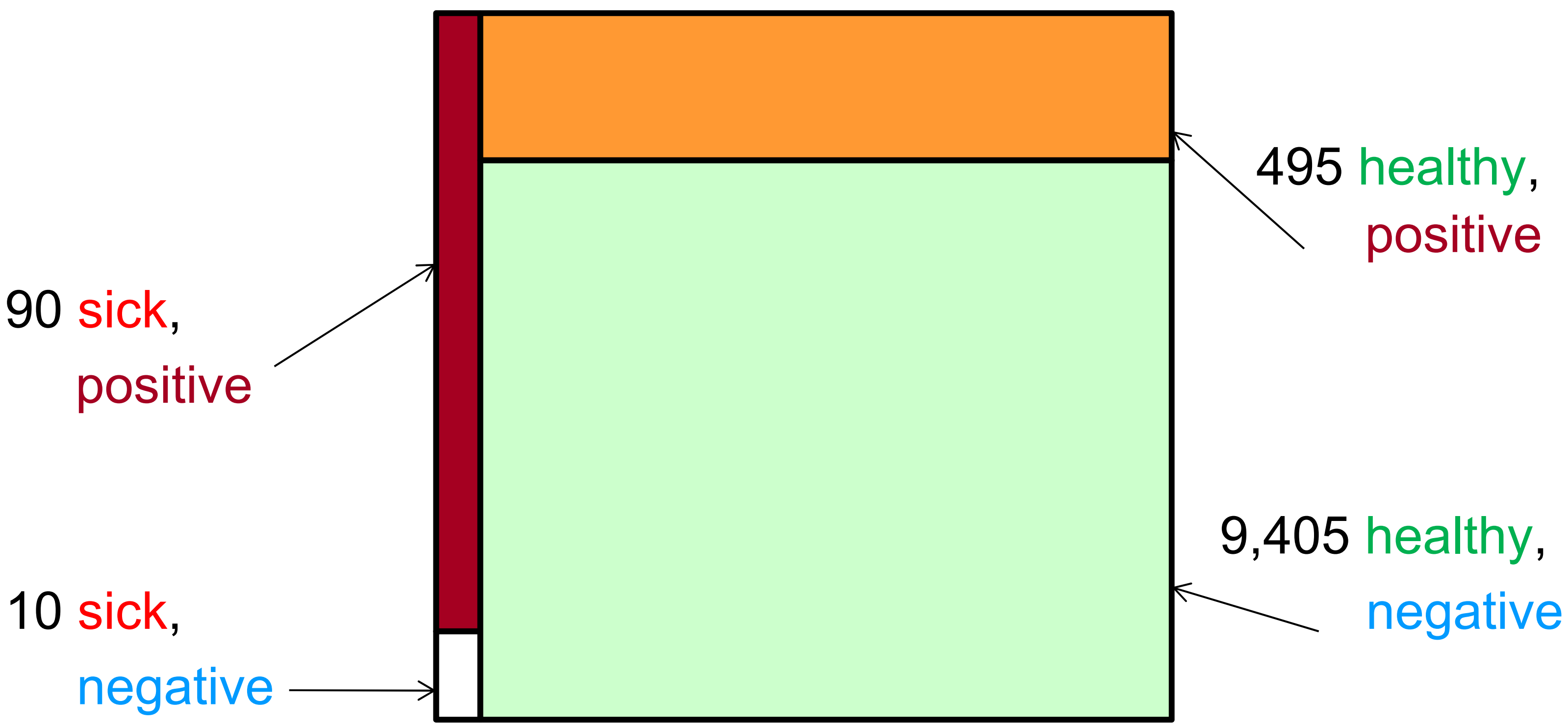
$$P(D \mid T) < 50\%$$

says it's not the end of the world either

The frequency story



The frequency story cont.



Ignoring Base Probabilities

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(\mathrm{A})}$$

So

$$P(B|A)P(A) = P(A \cap B) = P(A|B)P(B)$$

and

$$P(B|A) = \frac{P(B)}{P(A)} P(A|B)$$

Ignoring Base Probabilities

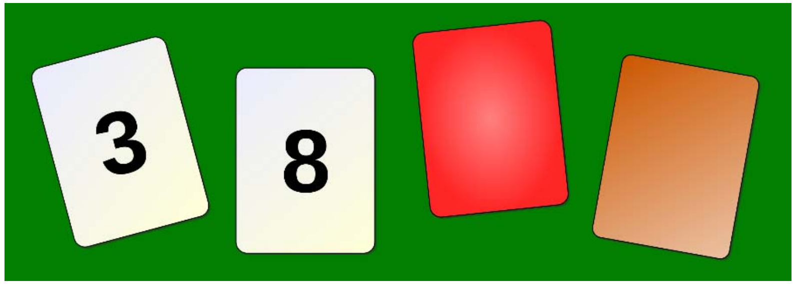
$$P(B|A) = \frac{P(B)}{P(\mathrm{A})} P(A|B)$$

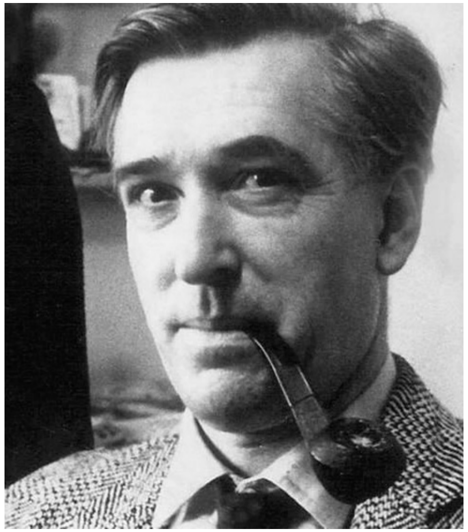
In general,

$$P(B|A) \neq P(A|B)$$

The deterministic analog

Wason selection task





Peter C. Wason (1924-2003)

Reasoning
Peter C. Wason (1966)
In Foss, B. M. (ed.). *New horizons in psychology* **1**
Harmondsworth: Penguin (1966)

Reference

Reasoning about a rule

Peter C. Wason

The Quarterly Journal of Experimental Psychology, Vol. 20 No. 3 (1968), pp. 273-281

Abstract

Two experiments were carried out to investigate the difficulty of making the contra-positive inference from conditional sentences of the form, “if P then Q.” This inference, that not-P follows from not-Q, requires the transformation of the information presented in the conditional sentence. It is suggested that the difficulty is due to a mental set for expecting a relation of truth, correspondence, or match to hold between sentences and states of affairs. The elicitation of the inference was not facilitated by attempting to induce two kinds of therapy designed to break this set. It is argued that the subjects did not give evidence of having acquired the characteristics of Piaget's “formal operational thought.”

Though…

The effect of experience on performance in Wason's selection task

James R. Cox, Richard A. Griggs

Memory and Cognition, Vol. 10 No. 5 (1982), pp. 496-502

Abstract

The Wason selection task is a hypothetico-deductive reasoning problem employing the logical rule of implication. Recent studies have indicated that performance on this task may be related to subjects' experience with the task content. Five versions of the task that differed in the manner in which they were related to the subjects' experience with a familiar implication relationship were examined. The correct solution rate varied as a function of both the subjects' extraexperimental and intraexperimental experience. A memory-cuing/reasoning-by-analogy explanation is proposed to account for the direct relationship between performance and the degree ofsimilarity to subjects' experience.

Why do we get confused?

It is true that

$$P(A \mid B) > P(A \mid \neg B)$$

is equivalent to

$$P(B \mid A) > P(B \mid \neg A)$$

Correlation is symmetric; but, generally,

$$P(B \mid A) \neq P(A \mid B)$$

Social prejudice

It is possible that:

Most top squash players are Pakistani

but

Most Pakistanis are not top squash players

(Pick your favorite prejudice…)

The gambling problem

You are going to play the roulette. You first sit there and observe, and you notice that the last five times it came up “Black.” Would you bet on “Red" or on “Black"?

The Gambler’s Fallacy

- If you believe that the roulette is fair, there is independence
- By definition, you can learn nothing from the past about the future
- Kahneman and Tversky: The law of large numbers doesn’t say that errors get corrected, they are simply diluted

The Law of Large Numbers (LLN)

Roughly, the average converges

to the expectation

Giving meaning to “expectation”

Jacob (James or Jacques) Bernoulli (1654 – 1705)

The LLN more formally

Let X_i be I.I.D. (Independently and Identically Distributed) and consider their average (obviously, also a random variable) :

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Then, with probability 1,

$$\bar{X}_n \rightarrow \mu = E(X_i)$$

when

$$n \rightarrow \infty$$

($\mu = E(X_i)$ is the expectation of each variable, hence also of the average)

LLN's assumptions

- If the variables are not independent, the theorem doesn't hold
  - Insuring houses in California against earthquakes
- If the variables are not identically distributed, the theorem doesn't hold (what would it say, exactly?)
  - Insurance for different levels of risk (and the Lemons problem)
- And if n is small, the theorem doesn't say much

Errors are "diluted"

Suppose we observe 1000 blacks

The prediction for the next $1{,}000{,}000$ will still be around

$$500{,}000\ ;\ 500{,}000$$

Resulting in

$$501{,}000\ ;\ 500{,}000$$

But

- It's very unlikely to get six Black's in a row…
- Indeed, the probability of $BBBBBB$ is $\left(\frac{1}{2}\right)^6 = \frac{1}{64}$
- But so is the probability of $BBBBBR$ – also $\left(\frac{1}{2}\right)^6 = \frac{1}{64}$

  As well as the probability of $BRBBRB$, $RBBRBR$…

Why do we get confused?

One possibility:

Confounding conditional and unconditional probabilities

- Thinking of $P(A)$ instead of $P(A|B)$
- The probability of two bombs on the plane is very low (ε^2) but it doesn't mean that it will help you to bring one yourself

Another effect

We use memory to assess likelihood

And then $BBBBBB$ is very special, in “a class of its own”

with low probability $\left(\frac{1}{2}\right)^6 = \frac{1}{64} = 1.5\%$

While $BRBRRB$, $RBBRBR$ may be lumped together

For example, the probability of 3 R out of 6 is

$$\binom{6}{3}\left(\frac{1}{2}\right)^6 = \frac{20}{64} = 31.25\%$$

Yet, to be honest

If the state lottery were to come up with $1,2,3,4,5,6$ (out of $1, \ldots, 47$) I'd be very surprised

(That's why I don't play the lottery)

I'd react differently to $1,2,3,4,5,6$ than to, say,

$$3,8,12,14,25,36$$

The Bulgarian State Lottery

- From a BBC news website (Sep 2009)

Bulgarian lottery repeat probed

The Bulgarian authorities have ordered an investigation after the same six numbers were drawn in two consecutive rounds of the national lottery.

The numbers - 4, 15, 23, 24, 35 and 42 - were chosen by a machine live on television on 6 and 10 September.

An official of the Bulgarian lottery said manipulation was impossible.

A mathematician said the chance of the same six numbers coming up twice in a row was one in four million. But he said coincidences did happen.

Why?

- Is it "rational to be surprised" by 1,2,3,4,5,6 ?

- Well, maybe. If there is an alternative theory to "pure randomness"

- If we're rational, we should probably always entertain some doubt about the data we were provided

Maybe the roulette is not fair?

- Indeed, we will have to conclude this after, say, $1{,}000{,}000$ Black's.

- But then we will expect Black, not Red

… It's hard to explain a preference for Red

Wait a minute…

- If the random variables are independent of each other, how do we learn?
- If we take a sample of, say, 100 X_i's, do we know more about X_{101}?
- If so – what happened to independence?
- If not – why are we taking the sample?

Two approaches to statistics

- Classical Statistics
  - There is no probability over unknown parameters
  - Probabilities are only over random variables given the unknown parameters
- Bayesian Statistics
  - There is probability over any unknown
  - If not objective – then subjective

Subjective Probabilities

Pascal, one of the founders of probability theory (if not *the* founder)

for games of chance ("risk")

suggested subjective probabilities and expected utility maximization in his "Wager"

Blaise Pascal (1623-1662)

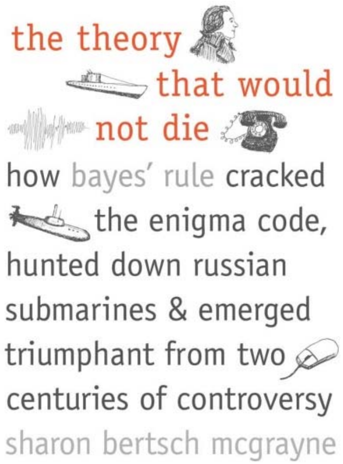
And then came Bayes

Bayesian updating is called after…

Rev. Thomas Bayes (1702-1761)

“Bayesian” – committed to having a subjective prior probability over any unknown

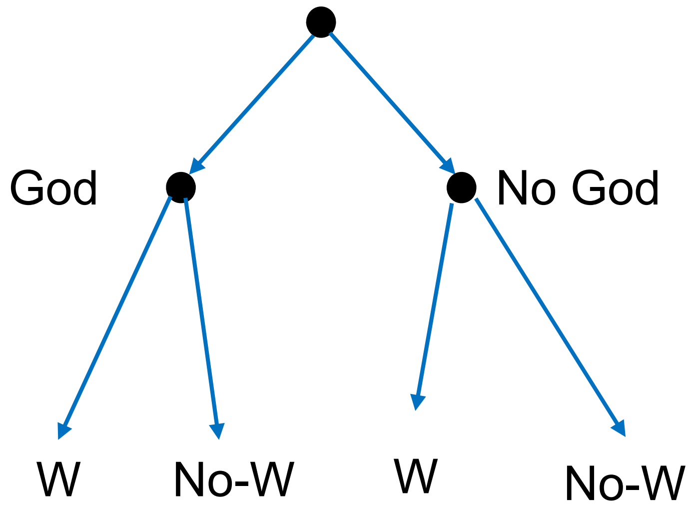
Relying on



Sharon B. McGrayne (b. 1942)

The existence of God

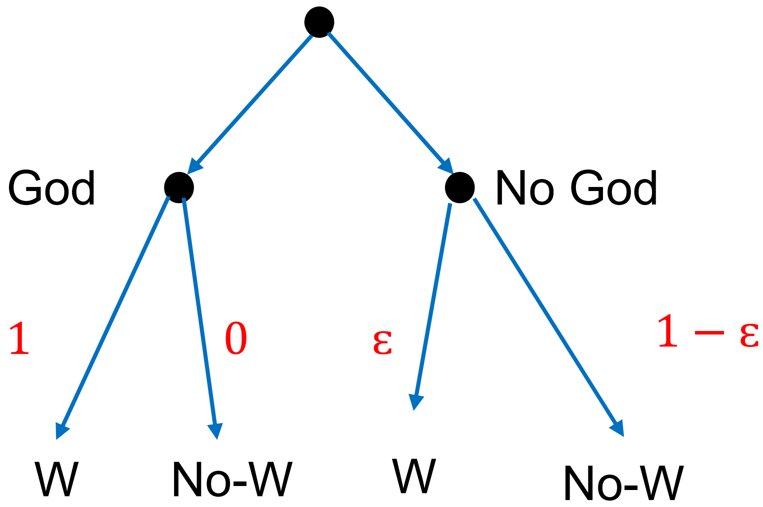
As described in McGrayne (2011), Bayes wanted to prove it:



The conditional probabilities

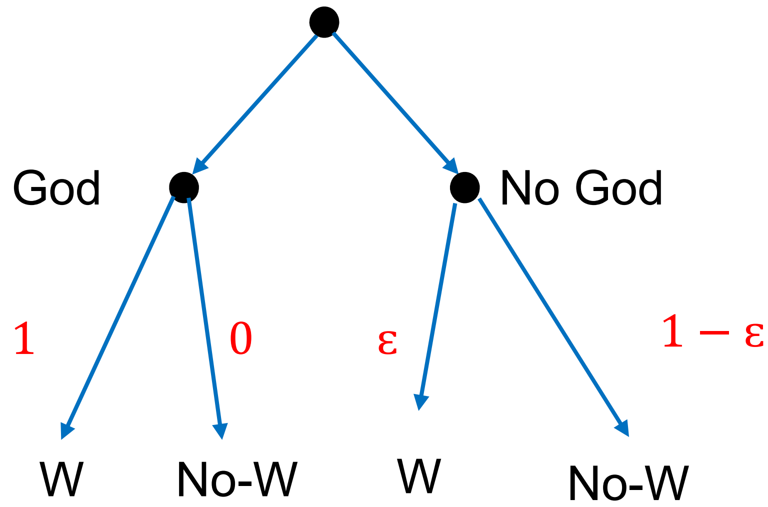
If God exists, we'll surely find the World as we know it

If not…

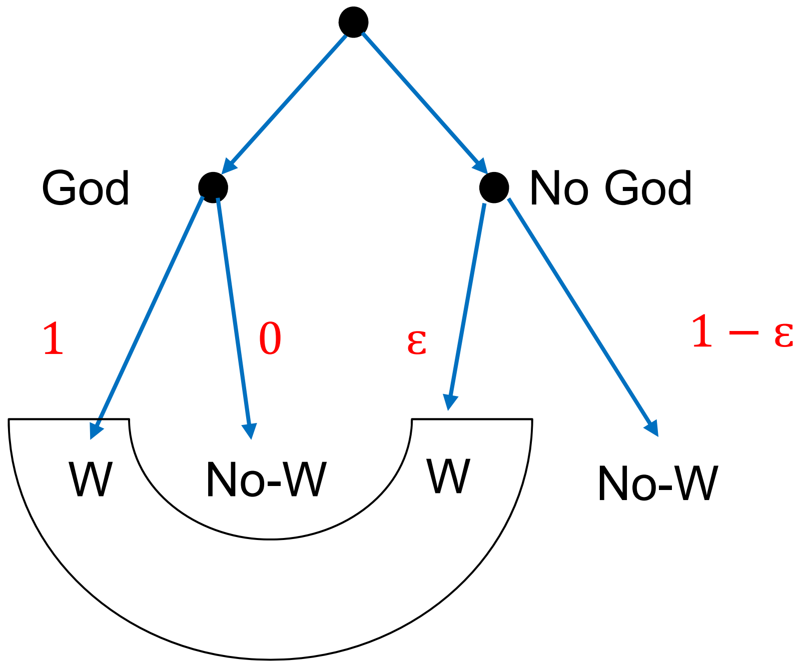


But we want the other direction…

$$P(\text{God}|\text{World}) \neq P(\text{World}|\text{God})$$



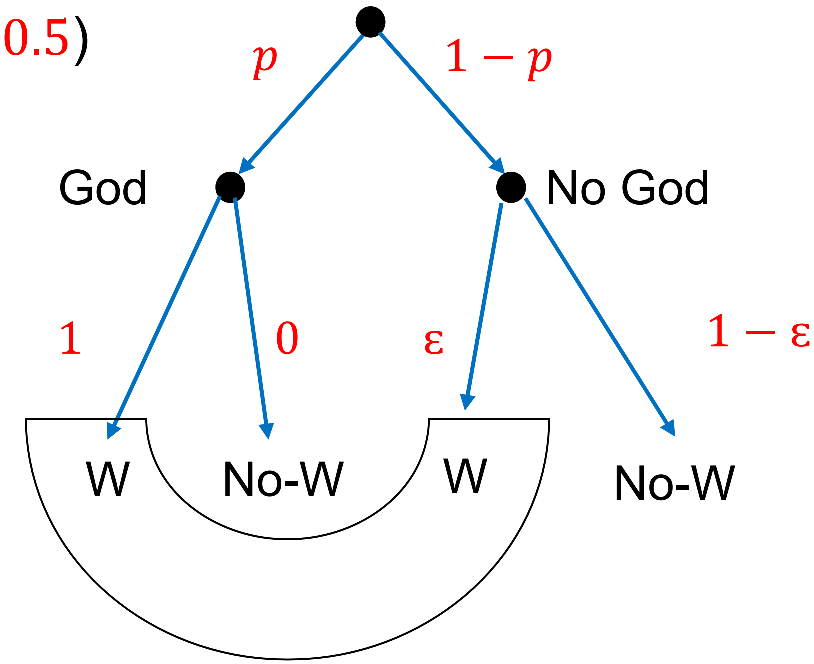
We need to collect probabilities



And we need a prior!

And then the argument can be completed

(Bayes used $p = 0.5$)



Time passed by

- The Bayesian approach is inherently subjective
- Which might be a reason it wasn't discussed much in the 19^{th} century
- But in the 20^{th}…
  - Bruno de Finetti (1931, 1937)
  - Bayesian Statistics
  - Big fights in the 1930-40s
  - More convergence today

Bayesian Statistics

- Tries to quantify subjective hunches, intuition

- Uses only probability

Bruno de Finetti (1906-1985)

Classical Statistics

- Trying to be objective
- Uses confidence intervals, hypotheses tests
- "Confidence", "significance" – not "probability"

Ronald A Fisher (1890-1962)

Back to our question

- If the random variables are independent from each other, how do we learn?
- If you're Classical statistician, you learn outside of the probability model
- If you're Bayesian, you don't have independence indeed
- You have only exchangeability (de Finetti)

Is the roulette fair?

- A Classical statistician will assume independence *given* the parameter of the roulette wheel, and will learn from observations about this parameter outside the probability model
- A Bayesian statistician will assume only conditional independence (*given* the parameter of the roulette wheel), and will learn about the parameter by Bayes's updating

Biased Samples

Problem

A study of students' grades in the US showed that immigrants had, on average, a higher grade point average than did US-born students. The conclusion was that Americans are not very smart, or at least do not work very hard, as compared to other nationalities.

Biased Samples

- The point: immigrants are not necessarily representative of the home population

- The Literary Digest 1936 fiasco
  - They predicted Alf Landon would beat Franklin Delanor Roosevelt 57% to 43%
  - As it turned out, Roosevelt won 62% to 37%

More biased samples

Everyday examples:

- Students who participate in class

- Citizens who exercise the right to vote

The Corona Virus

From the NY Times, February 13, 2020, an interview with Prof. Nicholls:

"… that just as with SARS there's probably much stricter guidelines in mainland China for a case to be considered positive. So the 20,000 cases in China is probably only the severe cases; the folks that actually went to the hospital and got tested. The Chinese healthcare system is very overwhelmed with all the tests going through. So my thinking is this is actually not as severe a disease as is being suggested. The fatality rate is probably only 0.8%-1%. There's a vast underreporting of cases in China.

Problem

In order to estimate the average number of children in a family, a researcher sampled children in a school, and asked them how many siblings they had. The answer, plus one, was averaged over all children in the sample to provide the desired estimate.

Inherently biased samples

Here the very sampling procedure introduces a bias.

A family of 8 children has 8 times higher chance of being sampled than a family of 1.

$$\frac{8 * 8 + 1 * 1}{9} = 7.22 > 4.5 = \frac{8 + 1}{9}$$

But

It is true that most children have many siblings

Whether a sample is biased or not depends on your question

How many poor families are there?

How many children grow in poverty?

Problem

A contractor of small renovation projects submits bids and competes for contracts. He noticed that he tends to lose money on the projects he runs. He started wondering how he can be so systematically wrong in his estimates.

The Winner's Curse

- Firms that won auctions tended to lose money
- Even if the estimate is unbiased ex-ante, it is not unbiased ex-post, given that one has won the auction.
- If you won the auction, it is more likely that this was one of your over-estimates rather than one of your under-estimates.

The Winner's Curse – an example

Real worth of an oil field – an unknown μ

Two firms ask experts for estimates

Each one provides an unbiased random variable X_i

$(i = 1,2)$

The estimators are unbiased: $E(X_i) = \mu$

Say, X_i can be $\mu - 5, \mu, \mu - 5$ with equal probabilities

The example – cont.

Joint distribution (probabilities)

X_1 \ X_2	$\mu - 5$	μ	$\mu + 5$	
$\mu - 5$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
μ	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
$\mu + 5$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1

The example – cont.

Probability of Firm 1 winning the contract

X_1 \ X_2	$\mu - 5$	μ	$\mu + 5$	
$\mu - 5$	$\frac{1}{18}$	0	0	$\frac{1}{18}$
μ	$\frac{1}{9}$	$\frac{1}{18}$	0	$\frac{3}{18}$
$\mu + 5$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{5}{18}$
	$\frac{5}{18}$	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{1}{2}$

The example – cont.

Firm 1's payoff

X_1 \ X_2	$\mu - 5$	μ	$\mu + 5$
$\mu - 5$	$+5$	0	0
μ	0	0	0
$\mu + 5$	-5	-5	-5

The example – cont.

Expected profit for Firm 1

X_1 \ X_2	$\mu - 5$	μ	$\mu + 5$	
$\mu - 5$	$\frac{1}{18}$	0	0	$\frac{1}{18} * (+5)$
μ	$\frac{1}{9}$	$\frac{1}{18}$	0	$\frac{3}{18} * (0)$
$\mu + 5$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{5}{18} * (-5)$
				$-\frac{20}{18}$

The Winner's Curse – main point

- The auction introduces a bias
- Given the fact that you won, you're more likely to be on the higher side
- Just imagine there were 10 firms

Regression to the Mean

This restaurant

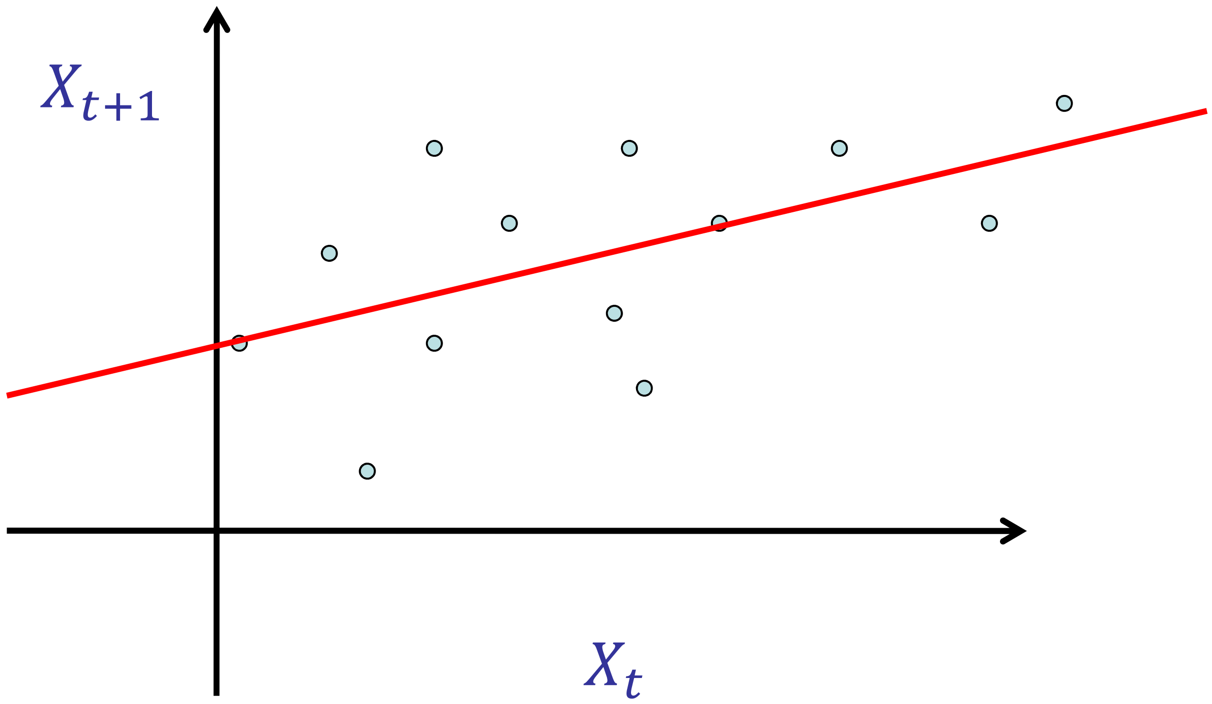
… isn’t as good as we recall it. Why?

- Maybe they don’t make an effort any longer
- Maybe we developed expectations

But also…

Regression to the Mean

Regressing $X_{t+1} = X_t + \varepsilon_t$



Why is called linear “regression”?

The method of least squares was invented by Legendre (1805) and Gauss (1809)

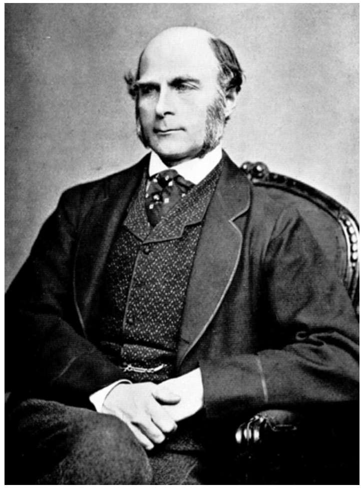
Carl Friedrich Gauss (1777-1855)

Adrien-Marie Legendre (1752-1833)

But “regression” is due to

Galton regressed the height of descendants on the height of parents

(Around the mid-1880s)



Francis Galton (1822-1911)

Regression to the Mean – main points

- We should expect an increasing line
- We should expect a slope < 1
- It need not follow a temporal or causal direction

… and in everyday life

- Students selected by grades
- Your friend's must-see movie
- The broker with the best performance over the past 5 years
- The best mayor

…

Causality

Problem

Studies show a high correlation between years of education and annual income. Thus, argued your teacher, it's good for you to study: the more you do, the more money you will make in the future.

Correlation and Causality

Possible reasons for correlation between X and Y:

- X is a cause of Y
- Y is a cause of X
- Z is a common cause of both X and Y
- Coincidence (should be taken care of by statistical significance)

Problem

In a recent study, it was found that people who did not smoke at all had more visits to their doctors than people who smoked a little bit. One researcher claimed, “Apparently, smoking is just like consuming red wine – too much of it is dangerous, but a little bit is actually good for your health!”

Correlation and Causality – cont.

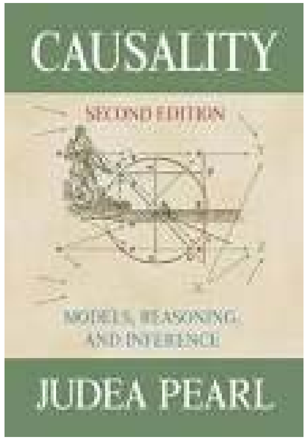
Other examples:

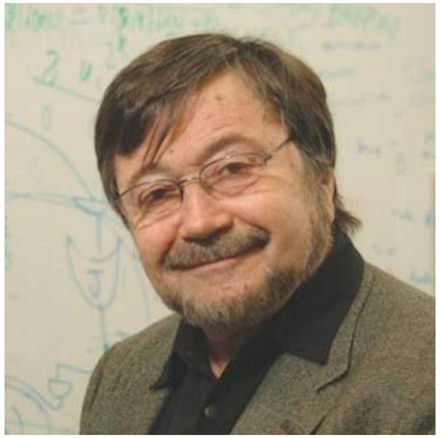
- Do hospitals make you sick?
- Will a larger hand improve the child's handwriting?

How to establish causality?

Major debates, as well as advancements

(Causality, 2000, The Book of Why, 2018)



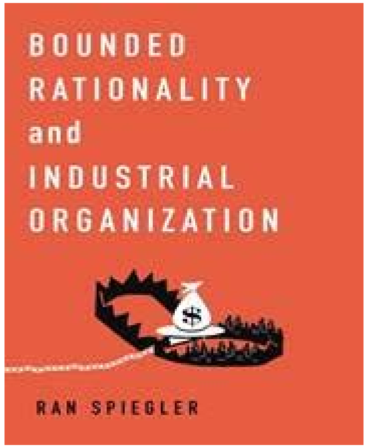


Judea Pearl (b. 1936)

Misperception of causality

Behavioral implications of causal misperceptions

Part of a larger project of IO implications of bounded rationality



Ran Spiegler (b. 1972)

Hypotheses Testing

The meaning of "significance"

…

Daniel: "Fine, it's your decision. But I tell you, the effects that were found were insignificant."

Charles: "Insignificant? They were significant at the 5% level!"

Statistical Significance

- Means that the null hypothesis can be rejected, knowing that, if it were true, the probability of being rejected is quite low
- Does not imply that the null hypothesis is wrong
- Does not even imply that the probability of the null hypothesis is low

The mindset of hypotheses testing

- We wish to prove a claim

- We state as the null hypothesis, H_0, its negation

- By rejection the negation, we will “prove” the claim

The mindset of hypotheses testing – cont.

- A test is a rule, saying when to say “reject” based on the sample

- Type I error: rejecting H_0 when it is, in fact, true

- Type II error: failing to reject H_0 when it is, in fact, false

The mindset of hypotheses testing – cont.

	H_0	H_1
Not reject	✓	Type II error
Reject	Type I error	✓

The mindset of hypotheses testing – cont.

- What is the probability of type I error?
  - Zero if the null hypothesis is false
  - Typically unknown if it is true
  - Overall, never known
- So what is the significance level, α ?
  - The maximal probability possible (over all values consistent with the null hypothesis)

The mindset of hypotheses testing – cont.

- We never state the probability of the null hypothesis being true
- Neither before nor after taking the sample
- This would depend on subjective judgment that we try to avoid

The mindset of hypotheses testing – Analogy

A court case

H_0 : The defendant is innocent

H_1 : The defendant is guilty

Asymmetry between the two

We give H_0 the benefit of the doubt

Acquittal does not mean a proof of innocence

The court case analogy

	H_0: innocent	H_1 : guilty
Not convict		Type II error
Convict	Type I error	

Consistency?

Would you join an experimental study of a drug that hasn't been approved yet?

In cases you're sure you would, why not just approve it?

Classical and Bayesian Statistics

- Bayesian:
  - Quantify everything probabilistically
  - Take a prior, observe data, update to a posterior
  - Can treat an unknown parameter, μ, and the sample, X, on equal ground
  - A priori beliefs about the unknown parameter are updated by Bayes rule

Classical and Bayesian Statistics – cont.

- Classical:
  - Probability exists only given the unknown parameter
  - There are no probabilistic beliefs about it
  - μ is a fixed number, though unknown
  - X is a random variable (known after the sample is taken)
  - Uses “confidence” and “significance”, which are not “probability”

Classical and Bayesian Statistics – cont.

- Why isn't "confidence" probability?
- Assume that $X \sim N(\mu, 1)$

$$Prob\,(\,|X - \mu| \leq 2\,) = 95\%$$

- Suppose

$$X = 4$$

- What is

$$Prob\,(\,2 \leq \mu \leq 6\,) = ?$$

Classical and Bayesian Statistics – cont.

- The question is ill-defined, because μ is not a random variable. Never has been, never will be
- The statement

$$Prob\ (|X - \mu| \leq 2) = 95\%$$

is a probability statement about X, not about μ

Classical and Bayesian Statistics – cont.

- If Y is the outcome of a roll of a die,

$$Prob\ (Y = 4) = \frac{1}{6}$$

- But we can't plug the value of Y into this, whether $Y = 4$ or not.
- Classical is complicated. Why use it?

A Bayesian approach to the court case

H_0 : The defendant is innocent

H_1 : The defendant is guilty

There is a prior probability on each (adding up to 1)

Evidence is gathered

The prior is updated to a posterior

Suppose the judge/jury is the defendant’s mom

Suppose they’re not…

Different methods for different goals

	Classical	Bayesian
Goal	To be objective	To express also subjective biases and intuition
For	Making statements in a society	Making the best decision for oneself
Analogous to	Rules of evidence	Self-help tool
To be used when you try	To make a point	To make a decision

Which is why we need both

- It's perfectly consistent to acquit a defendant, but not to want to ever see him again
- Or to join an experimental drug testing sample without approving it
- To use your intuition in looking for conjectures, but avoid it when proving them
- To do empirical research by Classical statistics, while assuming that agents are Bayesian

Having said that

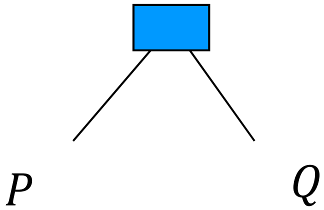
- Neither technique is perfect for its stated goal
- Classical statistics never achieves perfect objectivity
  - Objectivity is a direction, not a place
- Bayesian statistics may not be perfect in capturing our intuition
  - More on that later…

DECISION UNDER RISK

Expected Utility Theory

Problems 4.1 and 4.6

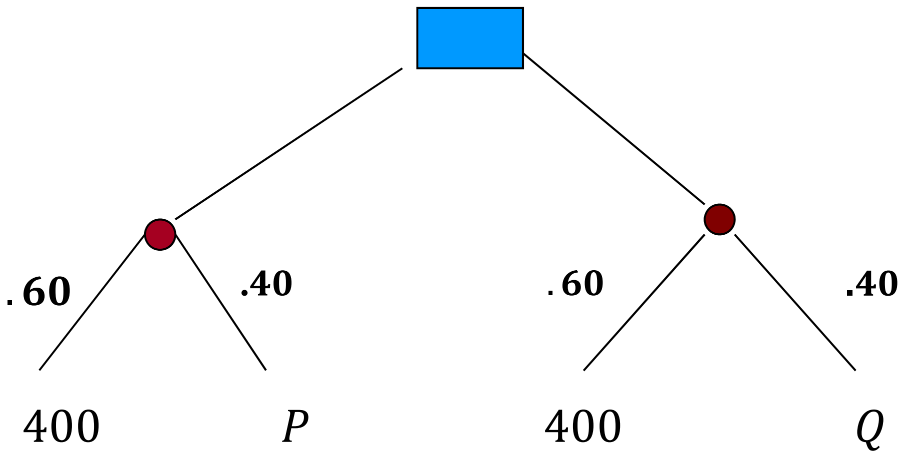
Problem 4.1



$$P = (.5, 0\ ;\ .5, 1000)$$

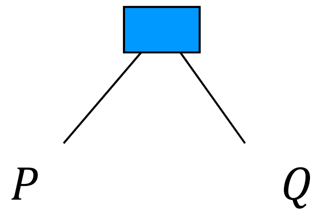
$$Q = (1{,}500)$$

Problem 4.6



Problems 4.2 and 4.7

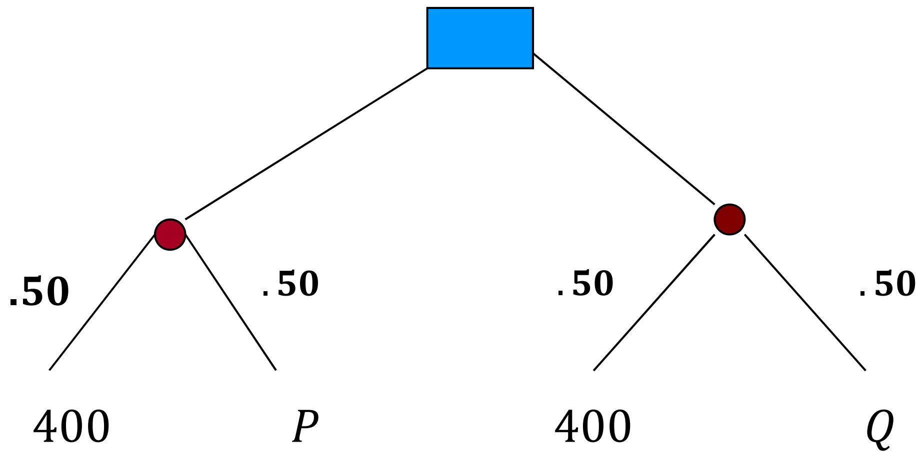
Problem 4.2



$$P = (.2, 0\ ;\ .8, 1000)$$

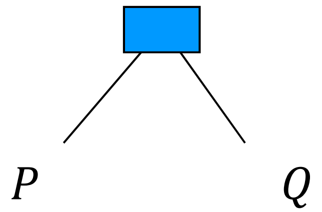
$$Q = (1{,}500)$$

Problem 4.7



Problems 4.3 and 4.8

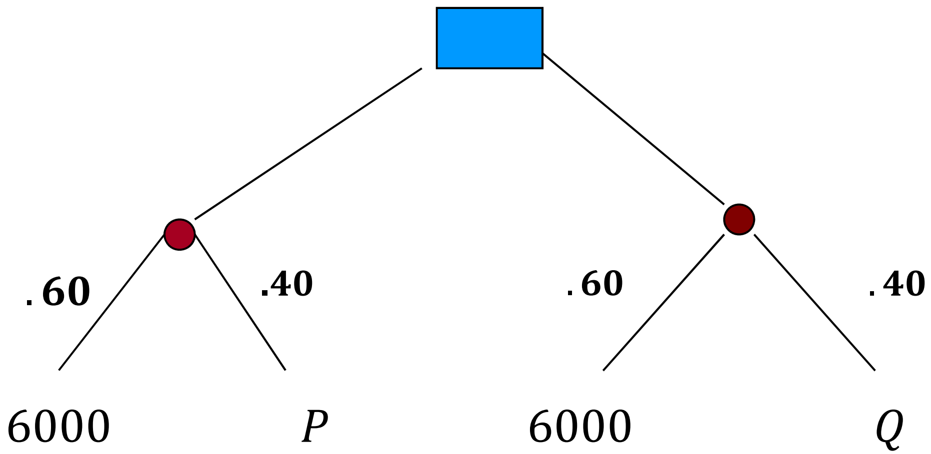
Problem 4.3



$$P = (.5, 2000\ ;\ .5, 4000)$$

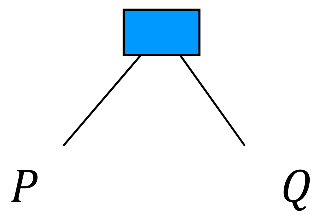
$$Q = (.5, 1000\ ;\ .5, 5000)$$

Problem 4.8



Problems 4.4 and 4.9

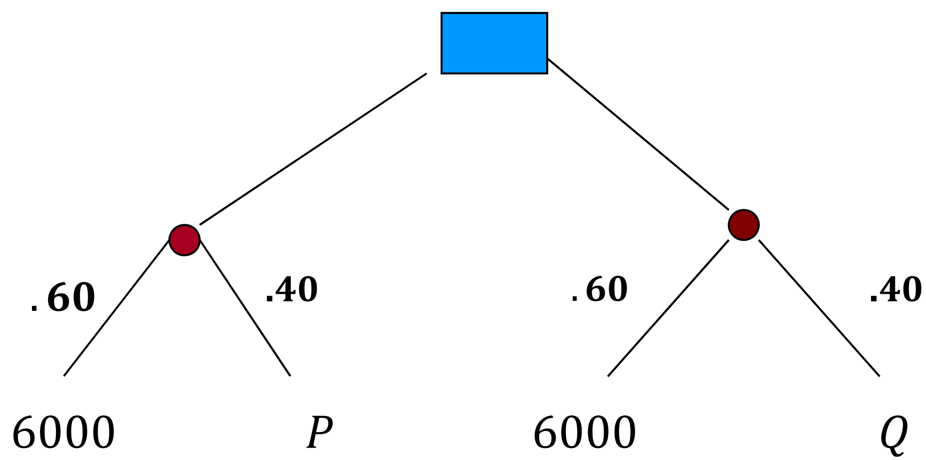
Problem 4.4



$$P = (.5, 2000\ ;\ .5, 4000)$$

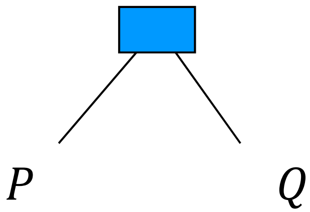
$$Q = (.4, 1000\ ;\ .6, 5000)$$

Problem 4.9

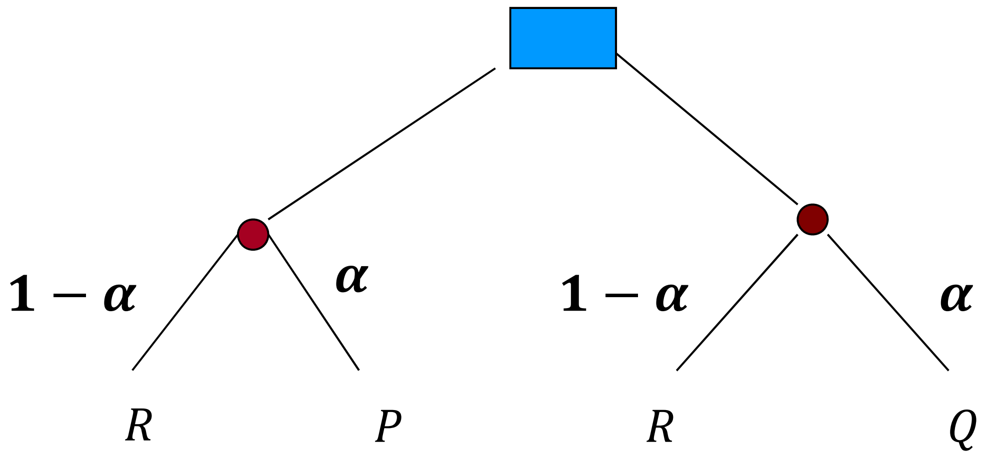


The Independence Axiom

The choice

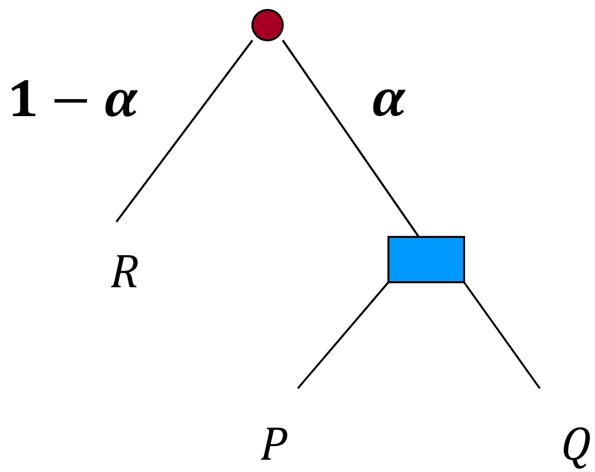


would/should be the same as

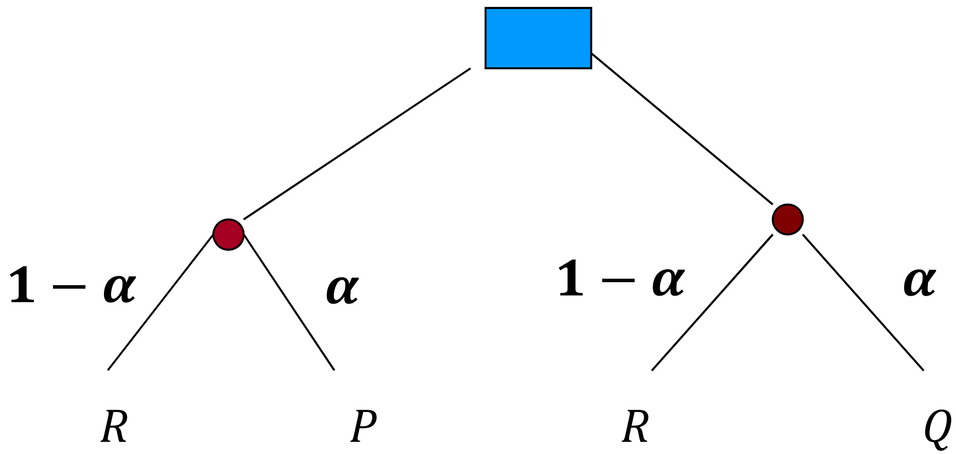


Assumption 1: Rational Planning

Compare your plan for choice in

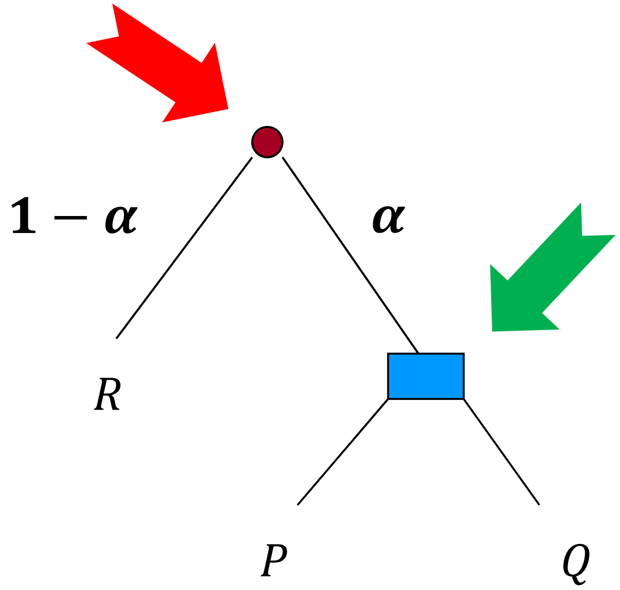


and your actual choice in



Assumption 2: Dynamic Consistency

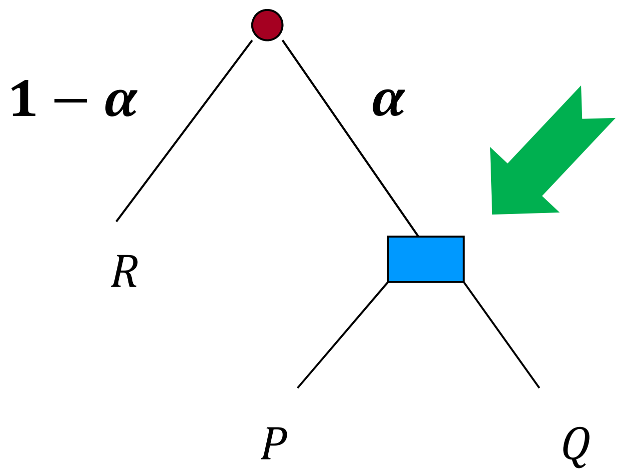
Your plan for choice in



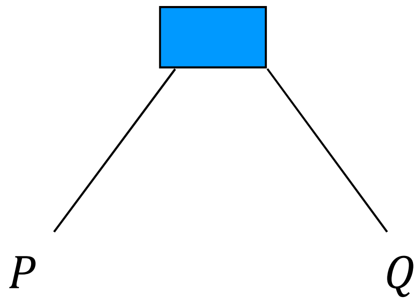
will indeed be followed

Assumption 3: Consequentialism

Your choice if and when you get to it in



will be the same as your choice in



Thus, Independence might not hold if

- We can’t plan rationally
  - Cognitive limitations: can one plan one’s reaction to an event such as Sep 11, 2001?
- We may not follow our plans
  - Emotional reactions: how would I respond to temptation? How would I deal with anger?
- We feel that the lotteries in the subtree aren’t the same as they would have been in a separate tree
  - History has an effect

The Independence Axiom as a formula

The preference between two lotteries P and Q is the same as between

$$(\alpha , P ; (1- \alpha) , R)$$

and

$$(\alpha , Q ; (1- \alpha) , R)$$

von-Neumann Morgenstern's Theorem

A preference order $\succcurlyeq$ over lotteries (with known probabilities) satisfies:

- Weak order (complete and transitive)
- Continuity
- Independence

IF AND ONLY IF

It can be represented by the maximization of the expectation of a "utility" function

Expected Utility

A lottery

$$(p_1, x_1; \; \ldots \; ; p_n, x_n)$$

is evaluated by the expectation of the utility:

$$p_1 * u(x_1) + \cdots + p_n * u(x_n)$$

Early origins: Pascal's "Wager"

	God is	God is not
Become a believer	∞	0
Forget about it		0

The basic argument: what have you got to lose?

What we call today "a (weakly) dominant strategy"

What Pascal didn't say

	God is	God is not
Become a believer	∞	0
Forget about it	$-\infty$	0

While some use burning in hell to scare you into faith, Pascal believed in positive marketing

Beyond dominance

	God is	God is not
Become a believer	∞	0
Forget about it		c

… But even if there is some $c > 0$ that you have to give up on by becoming a believer, it's finite.

Hence it's better to become a believer

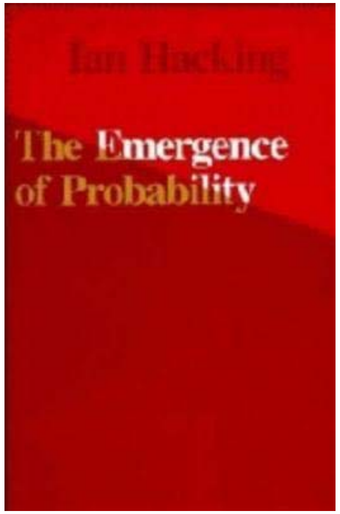
Pascal’s Wager – Ideas

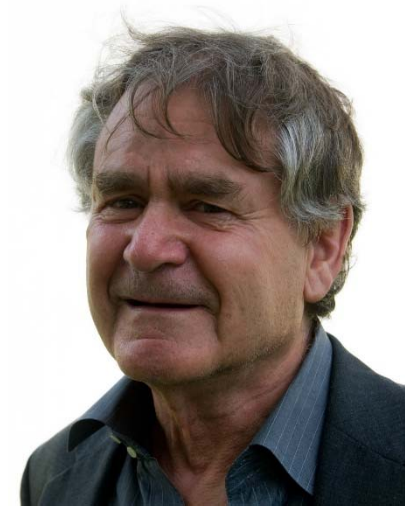
A few ideas in decision theory (Hacking, 1975)

- The decision matrix
- Dominance
- Subjective probability
- Expected utility
- Absence of probability

Not to mention humanism… (Connor, 2006)

The emergence of probability

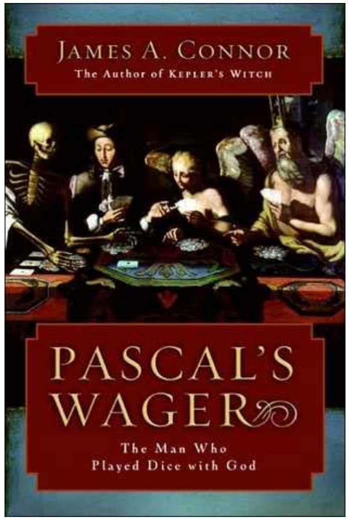




Ian Hacking (b. 1936)

Pascal’s Wager:

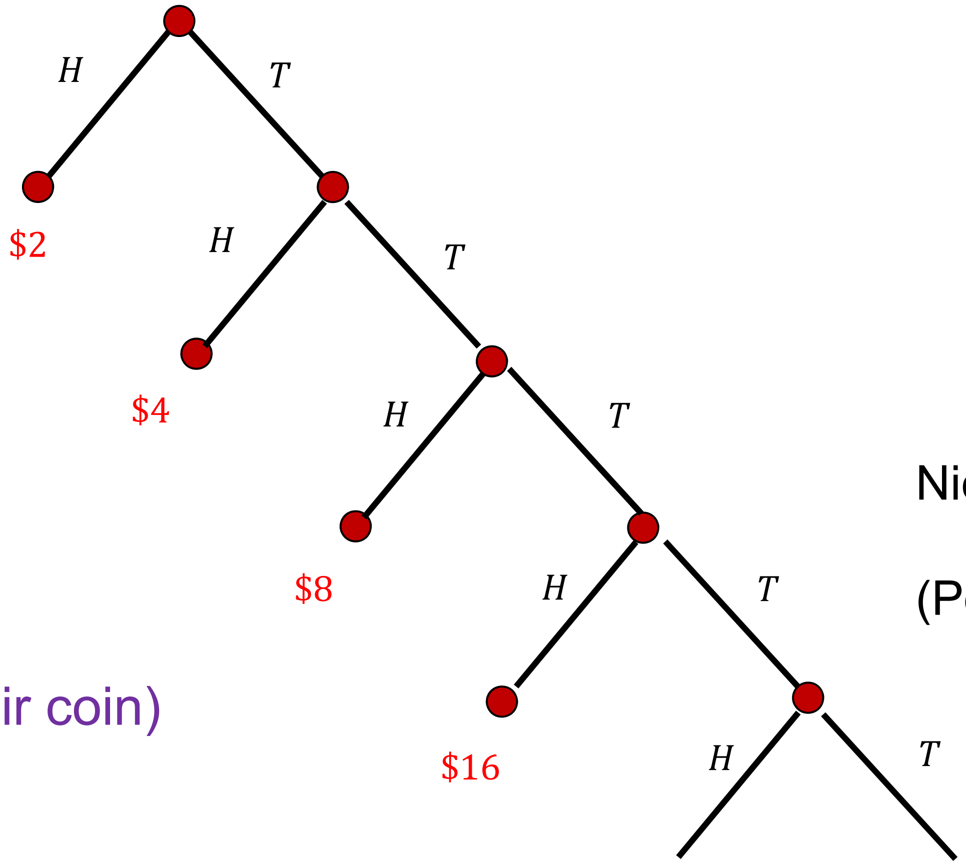
The man who played dice with God



James A. O’Connor

Next, a puzzle

How much would you pay to play:



(Fair coin)

Nicolaus Bernoulli (1687-1759)

(Posed the question in 1713)

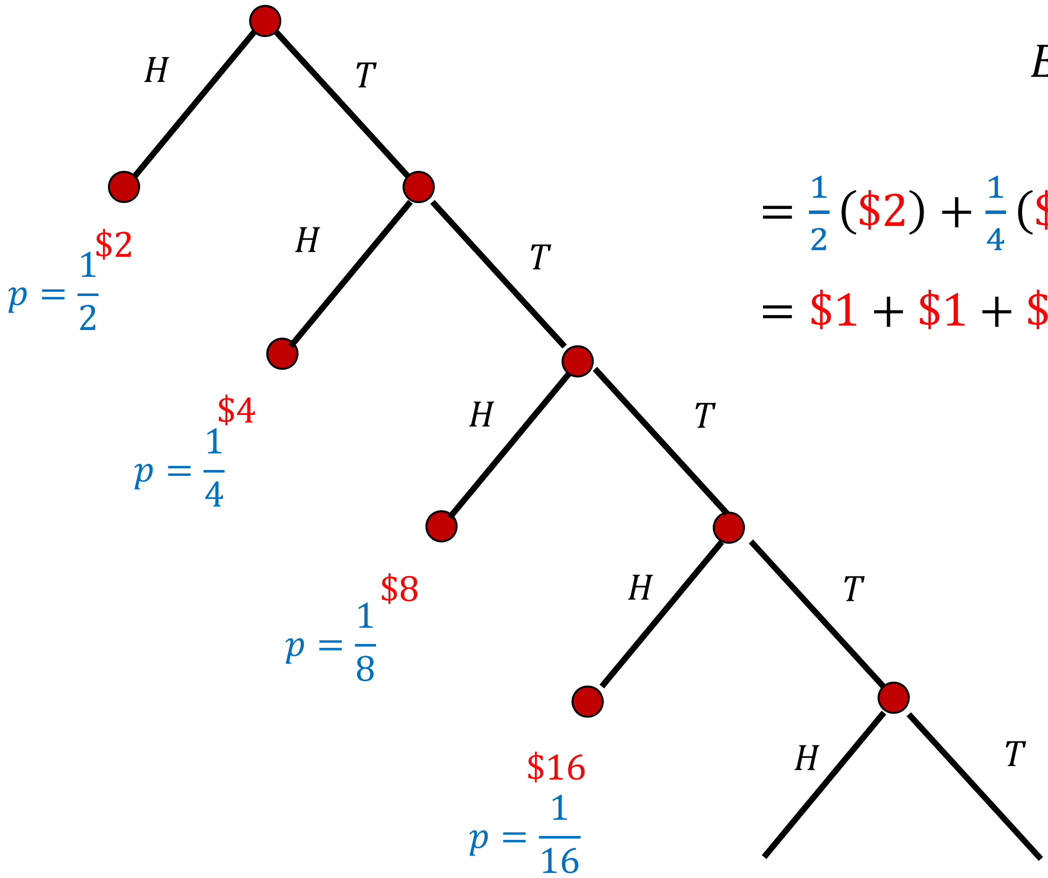
N. Bernoulli's game

- You only stand to gain
  - … at least $\$2$

- You know that a Tail will show up with probability 1
  - If something can happen with probability $\geq \varepsilon > 0$ every period, it eventually will

- True, we ignore time discounting

Expected profit in N. Bernoulli's game



$$EX = \sum p_i x_i$$

$$= \frac{1}{2}(\$2) + \frac{1}{4}(\$4) + \frac{1}{8}(\$8) + \frac{1}{16}(\$16) + \ldots$$

$$= \$1 + \$1 + \$1 + \$1 + \ldots$$

$$= \$\infty$$

"St. Petersburg Paradox"

- People aren't willing to pay any amount to play the game – despite the infinite expected value
- Daniel Bernoulli (1738): that's because they don't maximize expected value, but expected utility

Daniel Bernoulli (1700-1782)

Daniel Bernoulli's resolution

Instead of

$$EX = \sum p_i x_i = \frac{1}{2}(\$2) + \frac{1}{4}(\$4) + \frac{1}{8}(\$8) + \frac{1}{16}(\$16) + \cdots$$

Consider

$$Eu(X) = \sum p_i\, u(x_i) = \frac{1}{2}u(\$2) + \frac{1}{4}u(\$4) + \frac{1}{8}u(\$8) + \frac{1}{16}u(\$16) + \cdots$$

Daniel Bernoulli's intuition

The utility function is such that the marginal utility of money is inversely proportional to the amount of money we have

$$u'(x) = \frac{1}{x}$$

or (up to constants)

$$u(x) = ln(x)$$

and then

$$Eu(X) = \sum p_i \, ln(x_i)$$

$$= \frac{1}{2} ln(\$2) + \frac{1}{4} ln(\$4) + \frac{1}{8} ln(\$8) + \frac{1}{16} ln(\$16) + \cdots < \infty$$

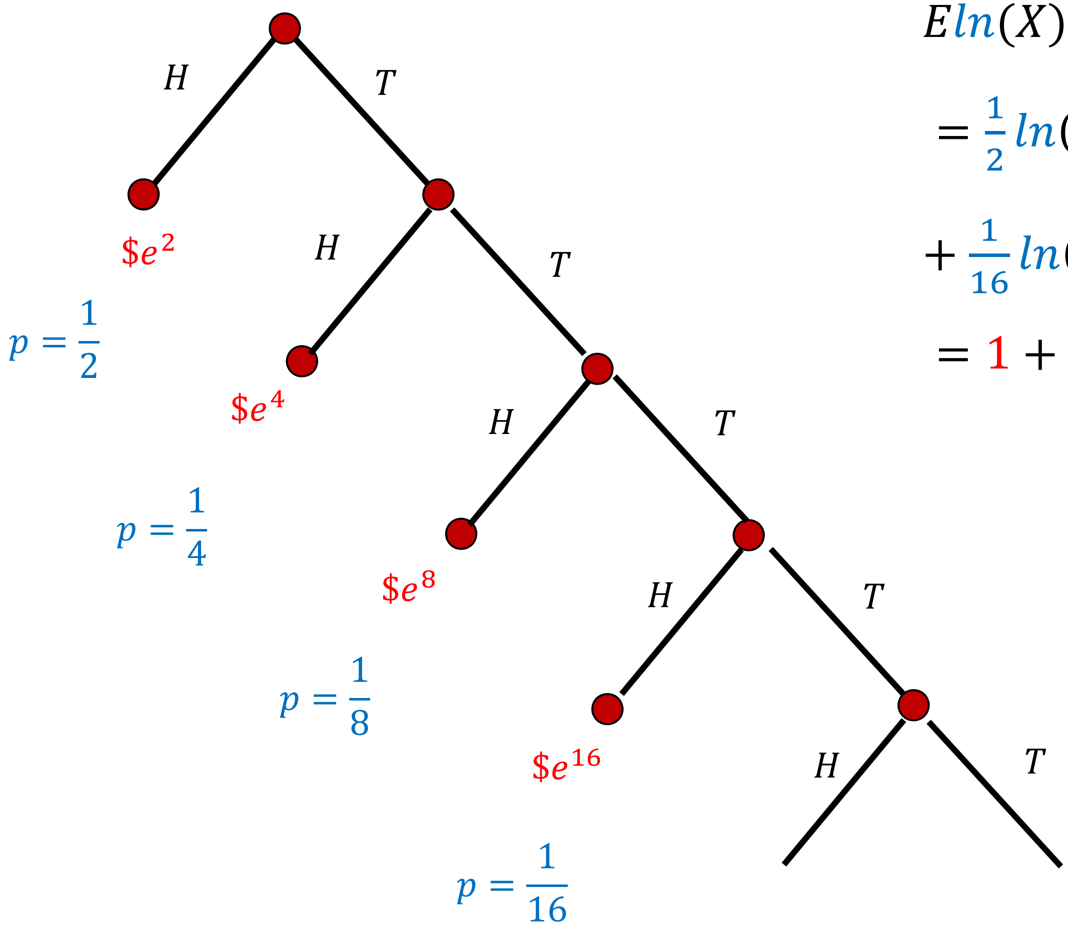
Reference

Exposition of a new theory on the measurement of risk

Daniel Bernoulli

Econometrica Vol. 22, No. 1 (Jan 1954), pp. 23-36

We could tease (D.) Bernoulli



$$E\,ln(X) = \sum p_i \, ln(x_i)$$

$$= \frac{1}{2} ln(\$e^2) + \frac{1}{4} ln(\$e^4) + \frac{1}{8} ln(\$e^8)$$

$$+ \frac{1}{16} ln(\$e^{16}) + \ldots$$

$$= 1 + 1 + 1 + 1 + \ldots$$

$$= \infty$$

In fact

- The paradox can be resurrected whenever the utility function is unbounded
- And it makes sense that the utility from money be bounded: at some point you will have bought the entire planet

Moreover, there is no paradox

- Even if my utility is unbounded, would I trust you to give me unbounded amounts of money?

- And why should I look at expected value to begin with?

- But let's not be petty. This is amazing.

Expected Utility Theory

Not much has happened in the next 200 years…

Until (the 1947, second addition of) "Games and Economic Behavior", the book in which von Neumann and Morgenstern more or less inaugurated game theory

John von Neumann (1903-1957)

Oskar Morgenstern (1902-1977)

von-Neumann Morgenstern's Theorem

When faced with choices between lotteries (with known probabilities) a decision maker satisfies

- Weak order
- Continuity
- Independence

IF AND ONLY IF

We can think of the decision maker as an EU maximizer

(Along the lines of the "as if" paradigm)

Let's understand the other axioms a bit better

Weak Order

– Preferences are complete:

For any two lotteries, P, Q there is preference

$$P \succcurlyeq Q, \text{ or } Q \succcurlyeq P$$

or perhaps both (indifference)

– Preferences are transitive:

For any three lotteries, P, Q, R,

$$\text{if } P \succcurlyeq Q, \text{ and } Q \succcurlyeq R, \text{ then } P \succcurlyeq R$$

Continuity

– For any three lotteries, P, Q, R,

if $P \succ Q \succ R$

then there are

$\alpha < 1$ and $\beta > 0$

such that

$$(\alpha, P; (1-\alpha), R) \succ Q \succ (\beta, P; (1-\beta), R)$$

Notice: continuity is in probabilities

(Not in outcomes, which may not even be numerical)

A counterexample to Continuity?

Suppose that

$$P - \$1 \qquad\qquad Q - \$0 \qquad\qquad R - Death$$

$$P \succ Q \succ R$$

Is there $\alpha < 1$ such that

$$(\alpha, P; (1-\alpha), R) \succ Q \quad ?$$

Will you risk your life for a dollar?

Will you cross the street to get a free newspaper?

Is Continuity reasonable?

Well, in terms of actual behavior – maybe so

(we do seem to take risks)

Perhaps more reasonable when dealing with less extreme outcomes

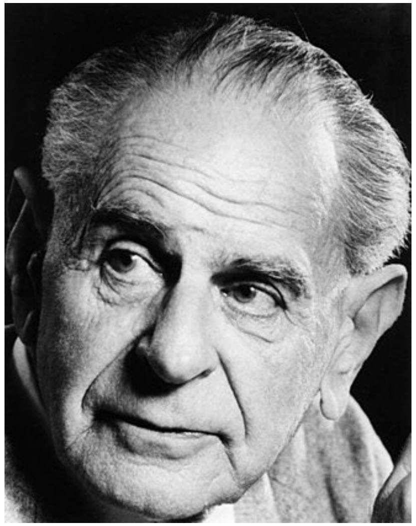
In any event, continuity is a matter of mathematical convenience

Continuity is irrefutable

No finite database could refute continuity

We can only test it in mind experiments

Popper would not allow us to use it



Karl Popper (1902-1994)

Back to vNM's Theorem

Very little is assumed, beyond Independence: we needed

Weak order

Continuity

Independence

to conclude that

behavior can be represented

by expected utility maximization

Implications of the Theorem

- Descriptively: It is perhaps reasonable that maximization of expected utility is a good model of people's behavior
  - Even if this isn't a description of a mental process they go through

- Normatively: Maybe we would like to maximize expected utility
  - Even if we don't do it anyway

“How unique” is the utility?

In consumer theory – any monotone transformation

$$v(x) = f\big(u(x)\big)$$

(for an increasing f) represents the same preferences

“utility is only ordinal”

Here – only affine transformations are allowed (but all of these) :

$$v(x) = au(x) + b$$

($a > 0$) – “utility is cardinal”

Why is the utility cardinal?

Why can't we have

$$v(x) = f\big(u(x)\big)$$

for a non-affine f?

- Because u doesn't only rank outcomes x
- It also ranks lotteries over x's via the expectation formula
- Non-linear transformations f would mess things up

But affine transformations are OK?

Yes. An affine transformation

$$v(x) = au(x) + b$$

commutes with expectation:

$$E[v(x)] = E[au(x) + b\ ]$$

$$= aE[u(x)] + b$$

… and maximizing Ev (when $a > 0$) is the same as

maximizing Eu

What does it mean?

Compare with temperature

$$v(x) = au(x) + b$$

This is the type of transformation between Celsius, Fahrenheit, and Kalvin

It doesn't mean much to say that the temperature is 5^o – on which scale?

The temperature analogy

So it is meaningless to say that the temperature is 5^o

Not even that it is positive

(It can be positive in Fahrenheit but not in Celsius)

(Does snow melt when it's 5^o ? – Depends…)

But it is meaningful to compare differences

“Tomorrow will be warmer than today by more than today is warmer than yesterday”

Similarly…

$$v(x) = au(x) + b$$

We don't attach any meaning to the numbers $u(x)$

Not even to their signs

But we can talk about differences: the comparison

$$u(x) - u(y) > u(y) - u(z)$$

will be meaningful

Cardinality – respect for differences

If

$$u(x) - u(y) > u(y) - u(z)$$

then for any

$$v(x) = au(x) + b$$

($a > 0$)

we also have

$$v(x) - v(y) > v(y) - v(z)$$

Because lotteries compare differences

If

$$u(x) - u(y) > u(y) - u(z)$$

then

$$\frac{1}{2}u(x) + \frac{1}{2}u(z) > u(y)$$

That is, the 50%-50% lottery between x and z is preferred to y

And this is an observation that v also has to describe

In other words

If both Eu and Ev represent preferences over lotteries, then the following are equivalent:

$$u(x) - u(y) > u(y) - u(z)$$

$$\frac{1}{2}u(x) + \frac{1}{2}u(z) > u(y)$$

the 50%-50% lottery between x and z is preferred to y

$$\frac{1}{2}v(x) + \frac{1}{2}v(z) > v(y)$$

$$v(x) - v(y) > v(y) - v(z)$$

And that would hold not only for $\left(\frac{1}{2};\frac{1}{2}\right)$ weights

The Independence Axiom – meaning

- It basically says that preferences are linear in probabilities – and that's what we'd expect…

$$p_1 * u(x_1) + \cdots + p_n * u(x_n)$$

- Not (necessarily) in outcomes
- The theory works also if the outcomes are not numerical at all (so that "linearity" isn't even defined)

Consider three outcomes

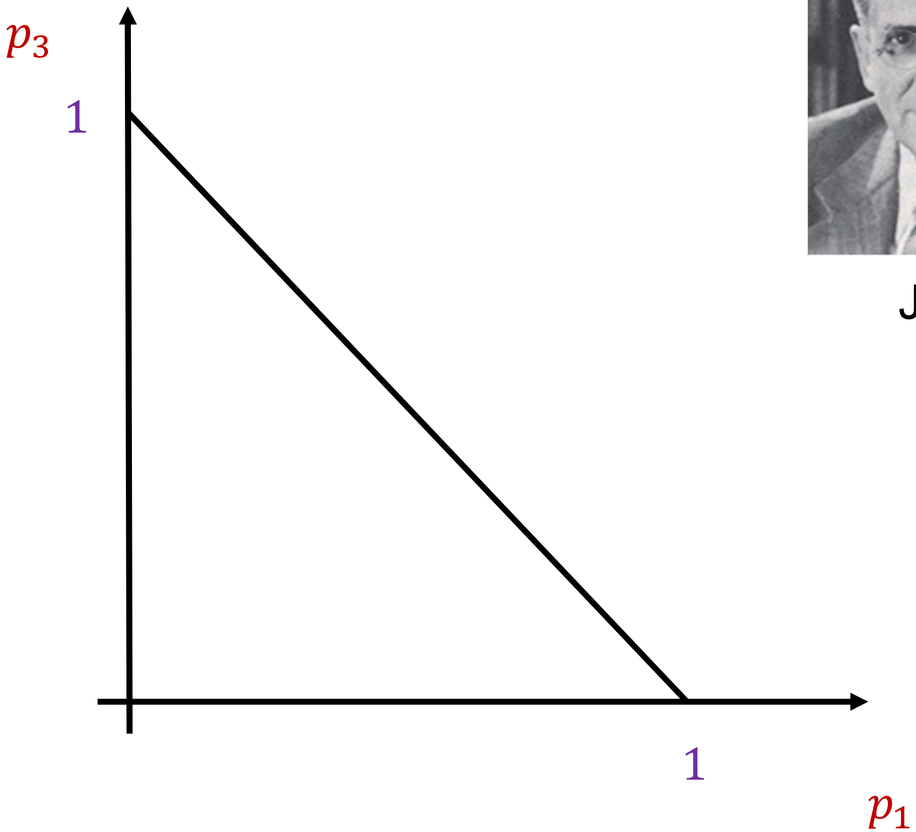
- Suppose that $x_1 \succ x_2 \succ x_3$
  - Where $\succ$ means strict preference
- Fix the three outcomes. A lottery

$$(p_1, x_1; p_2, x_2; p_3, x_3)$$

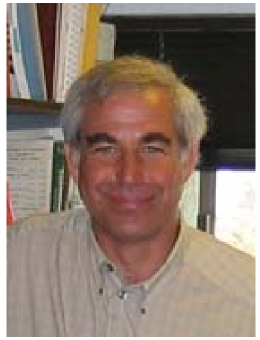
can be represented graphically by two numbers, p_1, p_3

where we recall that $p_2 = 1 - p_1 - p_3$

The Marschak-Machina Triangle

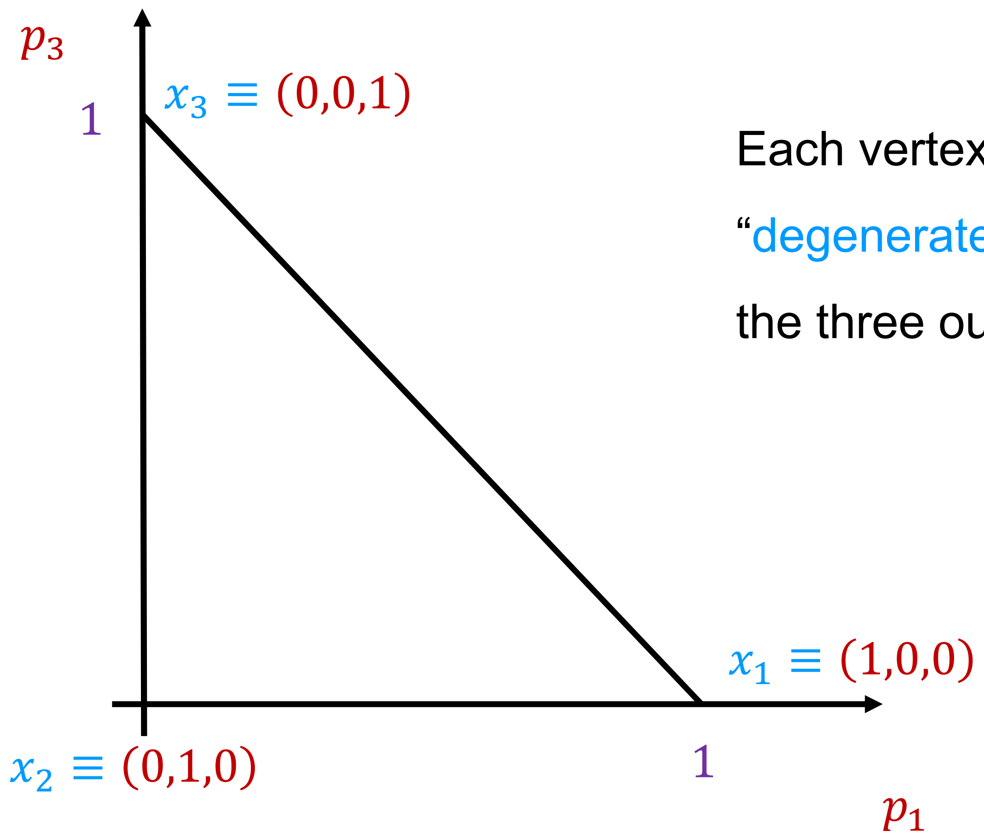


Jacob Marschak (1898-1977)



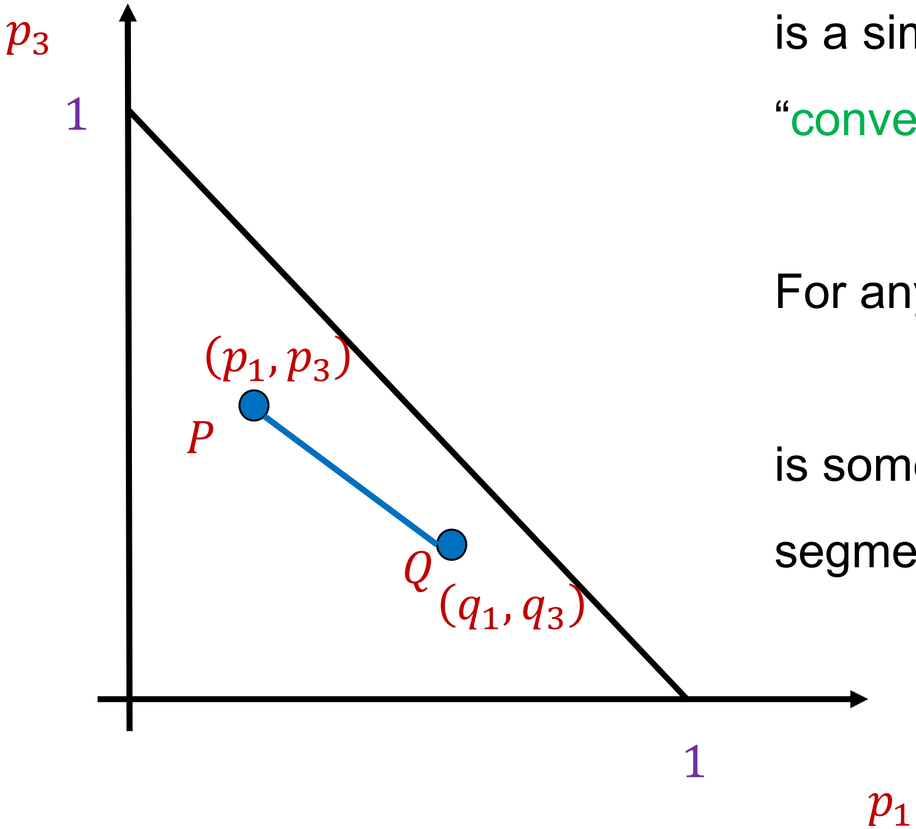
Mark J. Machina (b. 1954)

The Marschak-Machina Triangle



Each vertex corresponds to a "degenerate" lottery, yielding one of the three outcomes with certainty

The “mixture” operation



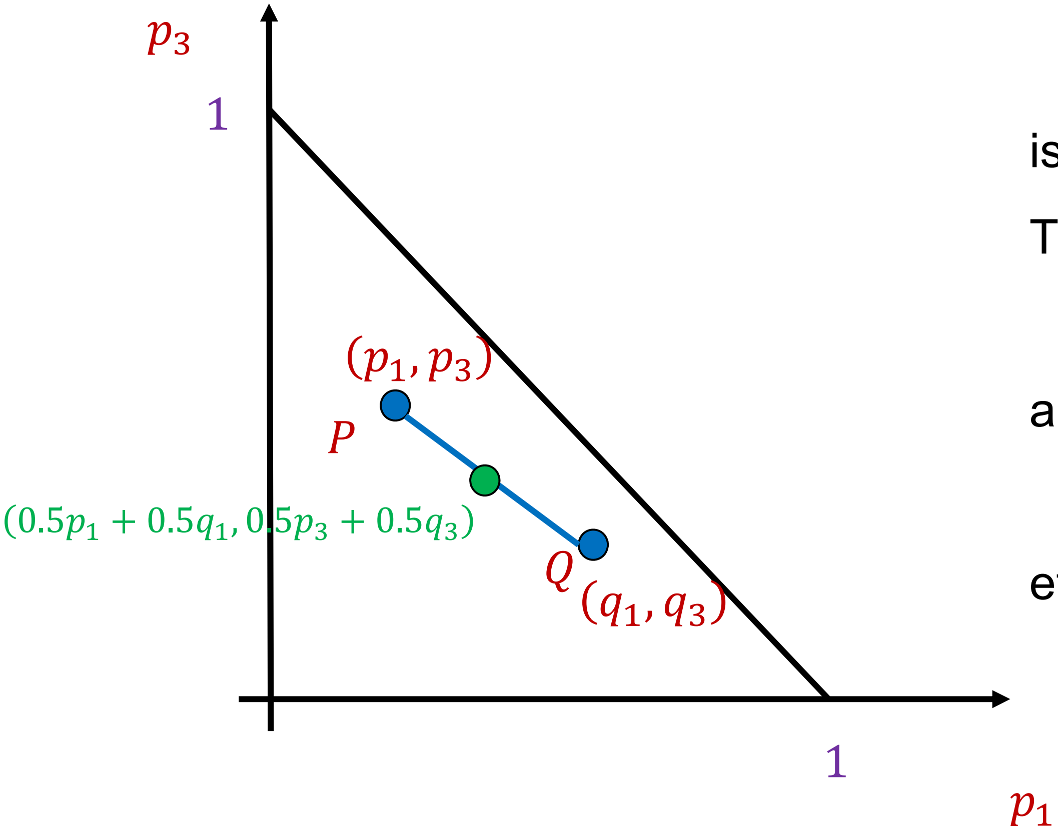
is a simple weighted average (or “convex combination”)

For any P , Q

$$(\ \alpha\ , P\ ;\ (1 -\ \alpha)\ , Q\ )$$

is somewhere on the straight line segment between P and Q

For example



For $\alpha = 0.5$

$$(\ 0.5\ , P\ ; 0.5\ , Q\ )$$

is the midpoint between P and Q

The probability to get x_1 is

$$0.5p_1 + 0.5q_1$$

and the probability to get x_3 is

$$0.5p_3 + 0.5q_3$$

etc. (for x_2)

First implication of the Independence Axiom

Indifference curves are linear:

If

$$P \sim Q$$

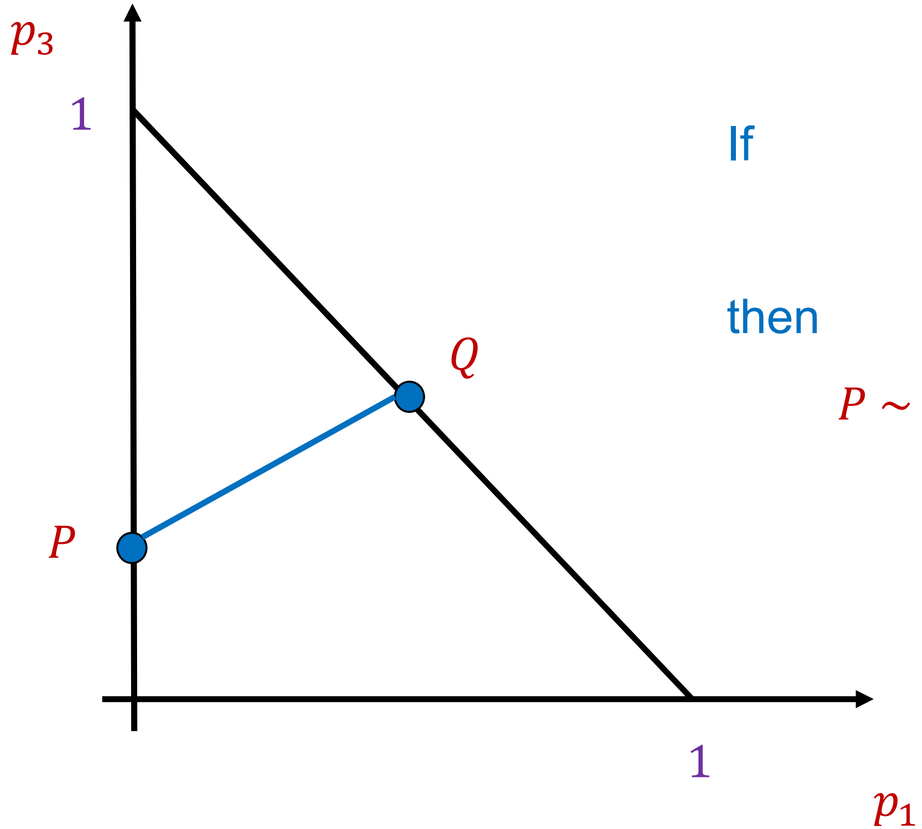
then

$$(\alpha, P; (1-\alpha), Q) \sim (\alpha, Q; (1-\alpha), Q) = Q$$

so that

$$P \sim (\alpha, P; (1-\alpha), Q) \sim Q$$

Linear indifference curves



If

$$P \sim Q$$

then

$$P \sim (\,\alpha\,, P\,;\,(1-\,\alpha)\,, Q\,) \sim Q$$

Second implication of the Independence Axiom

Indifference curves are parallel to each other:

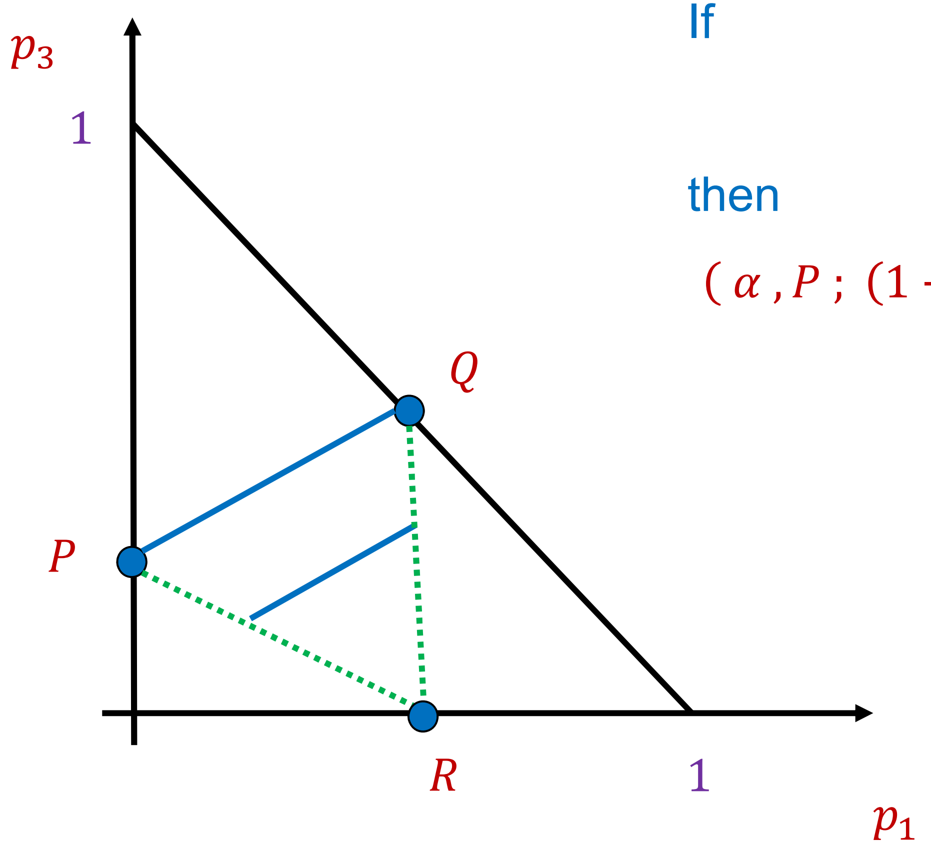
If

$$P \sim Q$$

then

$$(\alpha, P;\ (1-\alpha), R) \sim (\alpha, Q;\ (1-\alpha), R)$$

Indifference curves are parallel



If

$$P \sim Q$$

then

$$(\alpha, P;\ (1-\alpha), R) \sim (\alpha, Q;\ (1-\alpha), R)$$

(Thales's Theorem)

We could have

The Independence axiom limited to

If

$$P \sim Q$$

then

$$P \sim (\alpha, P; (1-\alpha), Q) \sim Q$$

Eddie Dekel (b. 1958)

Dekel's "betweenness"

Reference

An axiomatic characterization of preferences under uncertainty: weakening the independence axiom

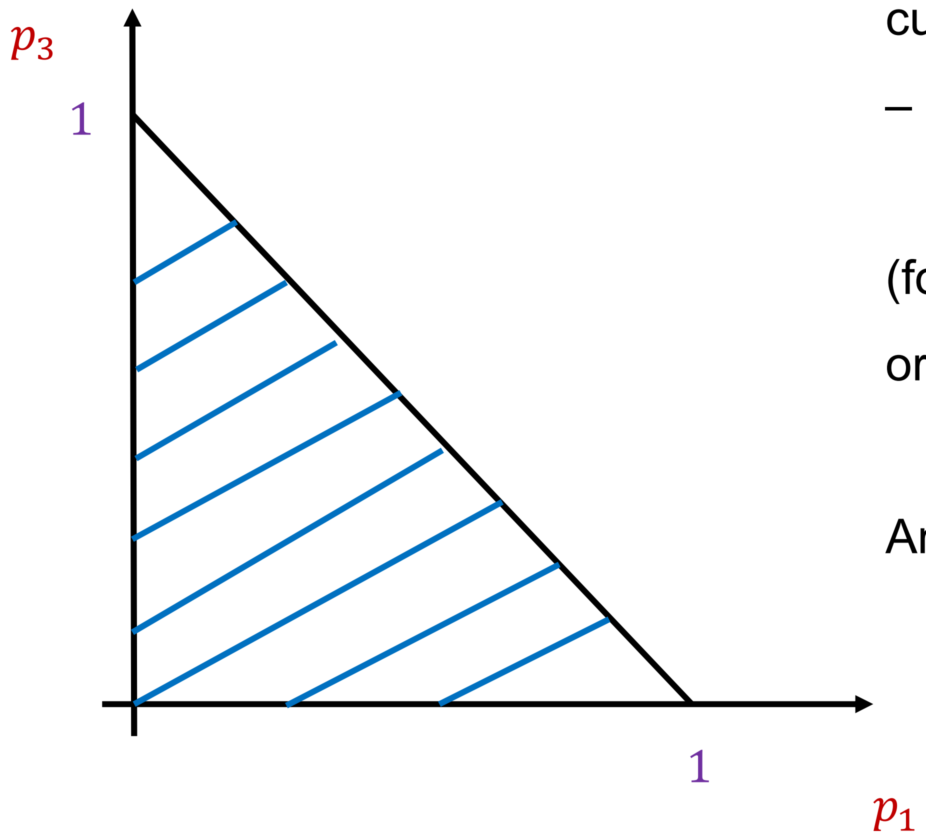
Eddie Dekel

Journal of Economic Theory, Vol. 40 (1986), pp. 304-318

Abstract

The independence axiom used to derive the expected utility representation of preferences over lotteries is replaced by requiring only convexity, in terms of probability mixtures, of indifference sets. Two axiomatic characterizations are proven, one for simple measures and the other continuous and for all probability measures. The representations are structurally similar to expected utility, and are unique up to a generalization of afhne transformations. First-order stochastic dominance and risk aversion are discussed using a method which finds an expected utility approximation to these preferences without requiring differentiability of the preference functional.

The Independence Axiom implies



Linear and parallel indifference curves

– all have the form

$$a_1 p_1 + a_3 p_3 = c$$

(for various c)

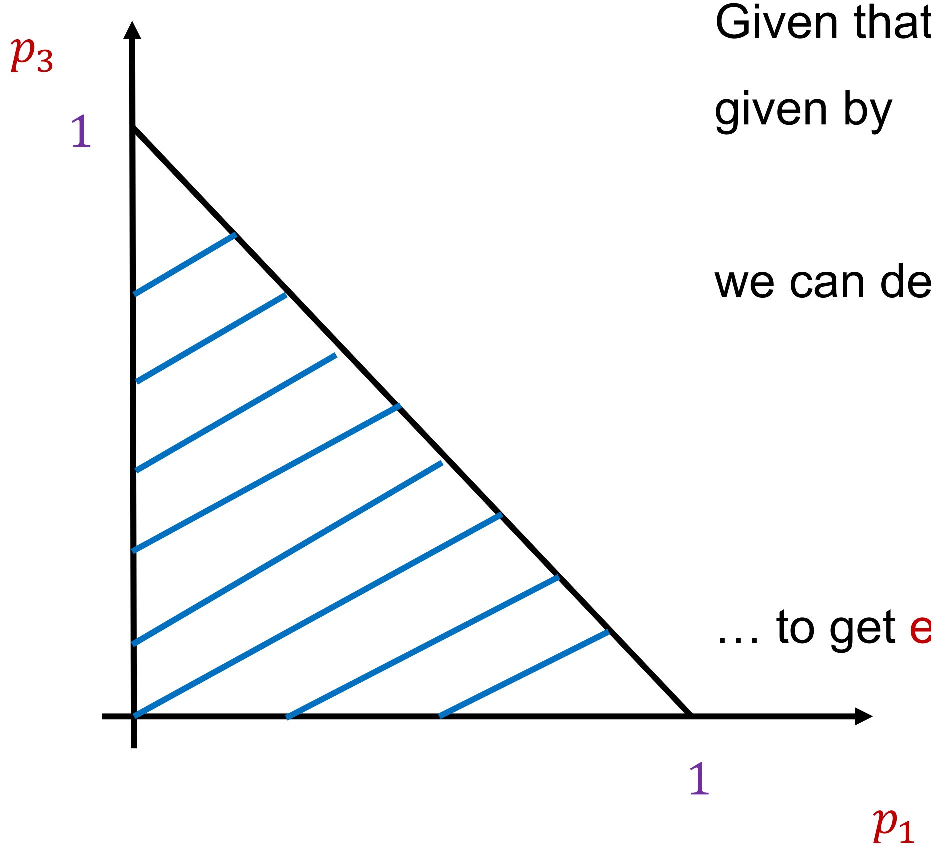
or

$$a_1 p_1 - c = -a_3 p_3$$

And we can assume

$$a_1 > 0 > a_3$$

The Independence Axiom implies



Given that the indifference curves are given by

$$a_1 p_1 + a_3 p_3 = c$$

we can define

$$u(x_1) = a_1$$

$$u(x_2) = 0$$

$$u(x_3) = a_3$$

… to get expected utility!

Calibration of utility

- If we believe that a decision maker is an EU maximizer, we can calibrate her utility function by asking, for which p is

$$(1, \$500)$$

equivalent to

$$((1-p), \$0 \,;\, p, \$1{,}000)$$

Calibration of utility – cont.

If, for instance,

$$u(\ \$0\ ) = 0$$

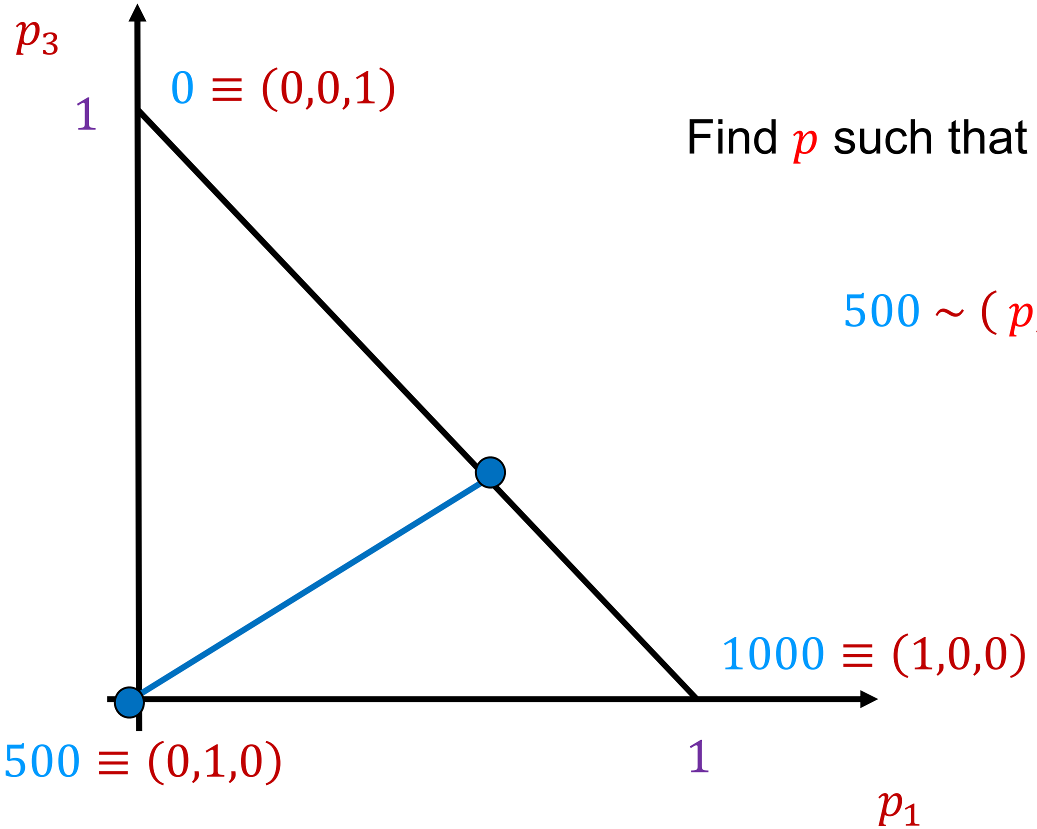
$$u(\ \$1{,}000\ ) = 1$$

Then

$$u(\ \$500\ ) = p$$

(And we can set two values of u as we wish – provided we respect monotonicity; the other values will be uniquely defined by preferences)

Calibration of utility in the Marschak-Machina Triangle



Find p such that

$$500 \sim (\,p, 1000\,;\ (1-p)\,, 0)$$

Risk aversion

- Do you prefer $\$500$ for sure or

$$(.50, \$0 \; ; \; .50, \$1{,}000) \text{ ?}$$

- Preferring $\$500$ for sure indicates risk aversion

- Risk aversion is defined as preferring $E(X)$ to X for every random variable X

Gambles

Let W be a given level of wealth

A bet is offered:

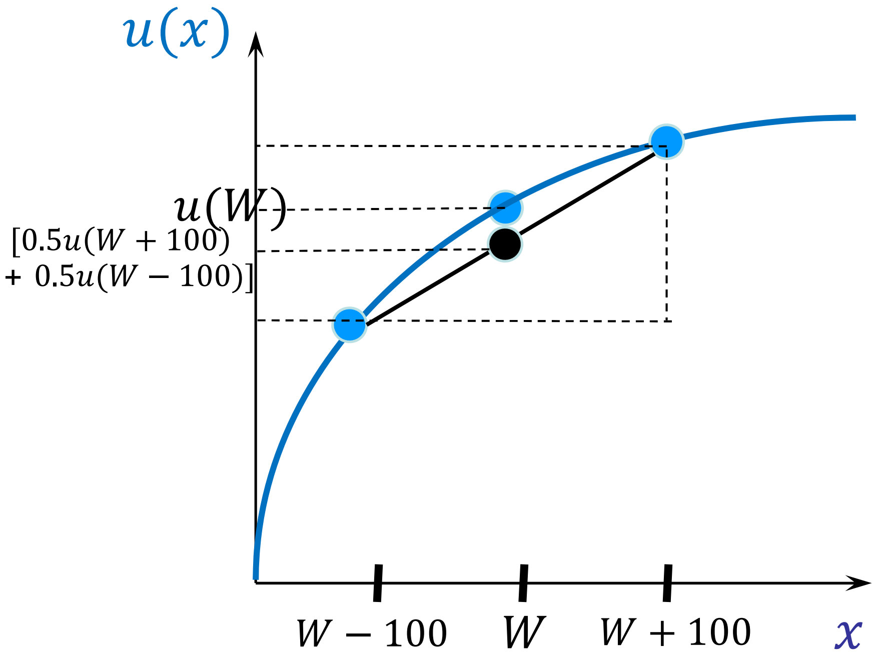
$$X = \begin{cases} +100 & .5 \\ -100 & .5 \end{cases}$$

It is a fair bet if

$$E(X) = 0 \qquad \text{or} \qquad E(W + X) = W$$

Expected value maximization implies indifference

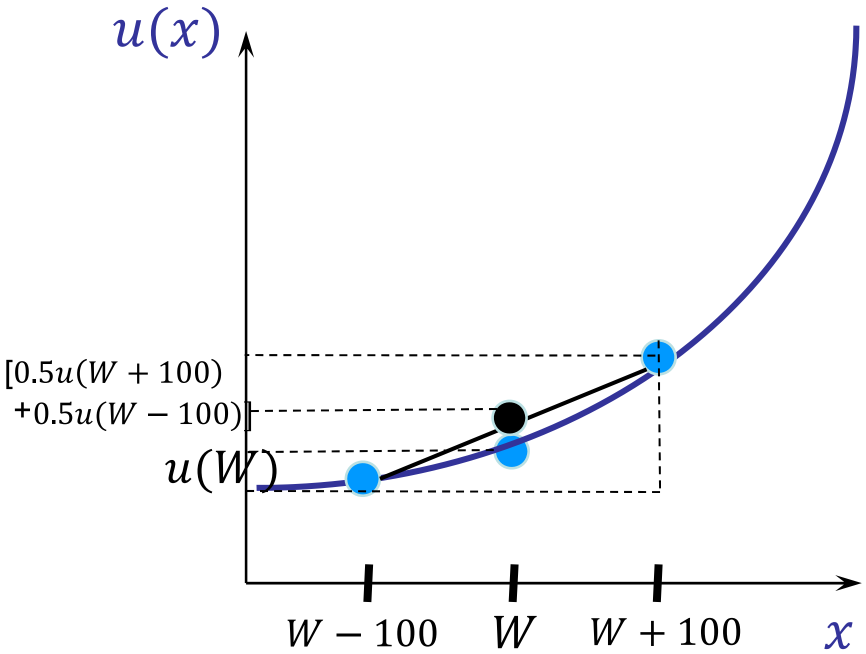
But…



If u is concave then

$$E\,u(W+X) \le u(W)$$

And, conversely for convex utility



If u is convex then

$$E\,u(W+X) \geq u(W)$$

Risk Aversion

… is defined as:

For every fair bet X $(E(X) = 0)$

and for every wealth level W

W is at least as desirable as $W + X$

Risk loving: just the opposite

Under EU maximization

Risk aversion $\Leftrightarrow u$ concave

Risk loving $\Leftrightarrow u$ convex

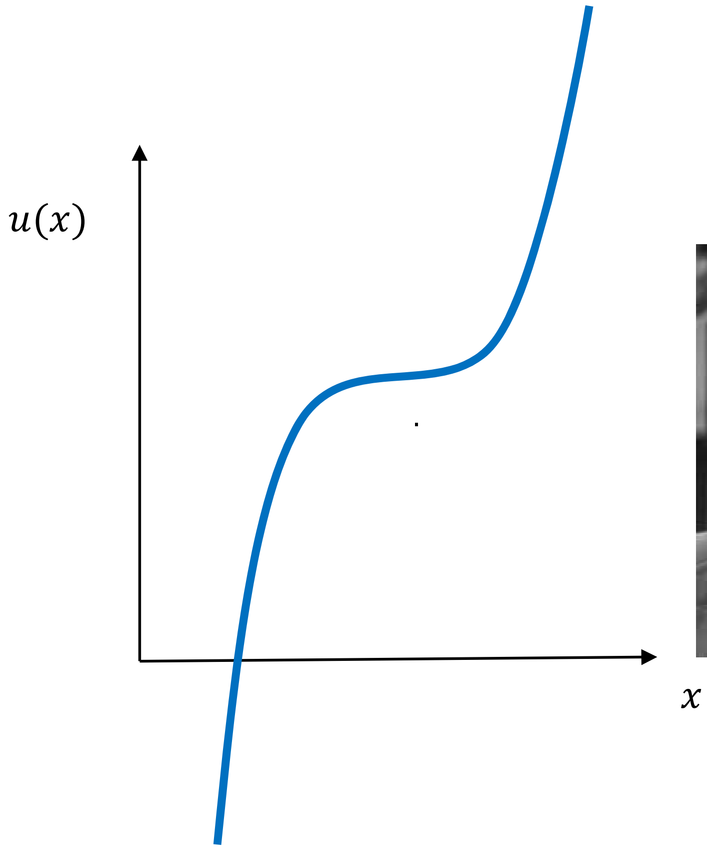
(Both can be defined in a strict sense as well)

Difficulties and Alternative Theories

EU maximization can explain

- Insurance (concave u)
- State lotteries (convex u)
- But… what about both occurring simultaneously?

An attempt to explain both

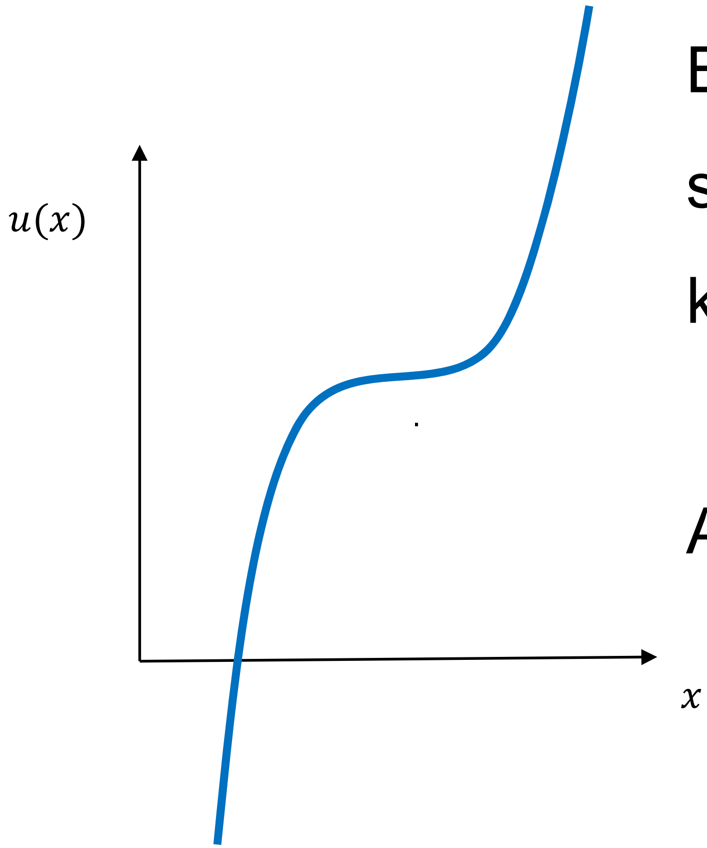


Maybe u starts out concave and turns to be convex?

Milton Friedman
(1912-2006)

Leonard Jimmie Savage
(1917-1971)

Well…



But then if one gets rich one should stop buying insurance and keep buying lottery tickets

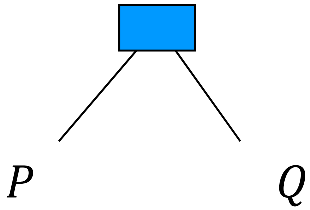
And that's not what we observe

Problems 4.5 and 4.10

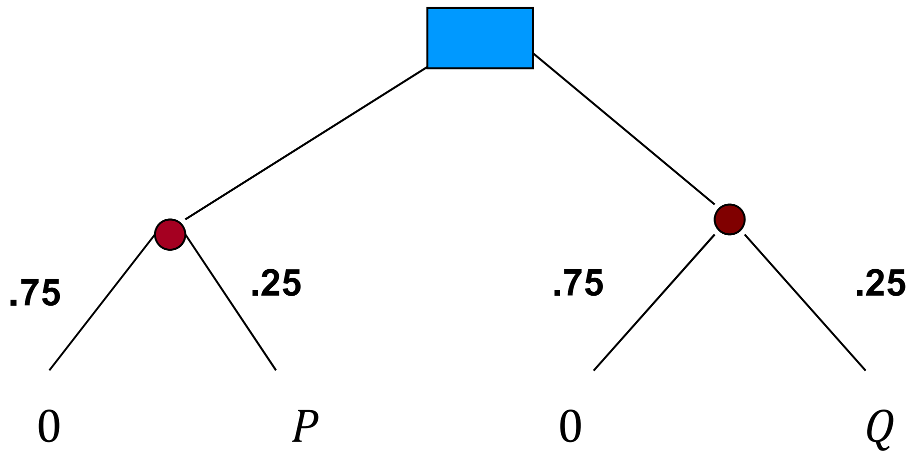
Problem 4.5

$$P = (.2, 0\ ;\ .8, 4000)$$

$$Q = (1, 3000)$$



Problem 4.10



Ample evidence that

- The independence axiom fails in examples such as 4.5 and 4.10.

- A version of Allais's paradox

Allais's Paradox

Which do you prefer?

Prob	Outcome
1.00	**$\$1M$**

vs.

Prob	Outcome
0.01	$\$0$
0.89	$\$1M$
0.10	$\$5M$

Maurice Allais (1911-2010)

And

Prob	Outcome
0.89	**$\$0$**
0.11	**$\$1M$**

vs.

Prob	Outcome
0.90	$\$0$
0.10	$\$5M$

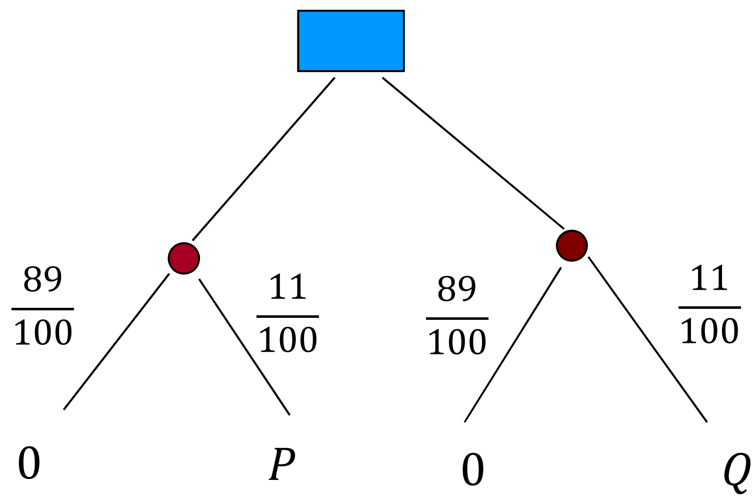
Reference

Le comportement de l’homme rationnel devant le risque: critique des postulats et axiomes de l’ecole americaine

Maurice Allais

Econometrica, Vol. 21, No. 4 (Oct. 1953), pp. 503-546

Allais's Paradox in decision trees



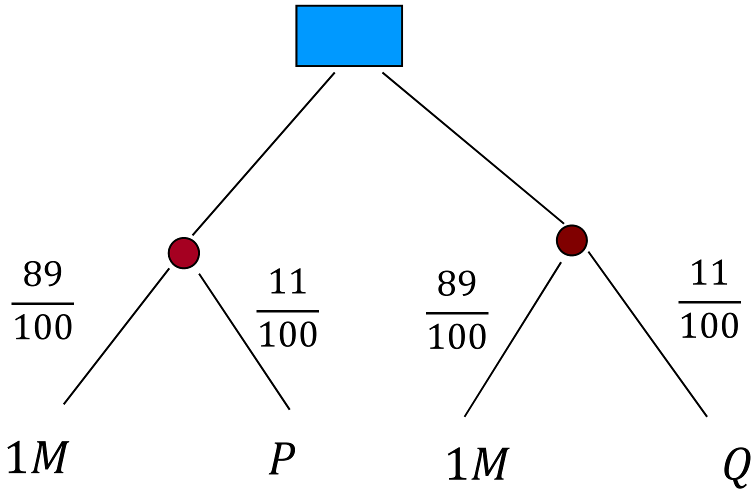
$(.90\,,\ 0\ ;\ .10\,,\ 5M)$ $\quad$ $(.89\,,\ 0\,;\ .11\,,1M)$

$$P = (\frac{1}{11}, 0\,; \frac{10}{11}, 5M)$$

$$Q = (1, 1M)$$

First choice

Allais's Paradox in trees – cont.



$$P = (\frac{1}{11}, 0\,; \frac{10}{11}, 5M)$$

$$Q = (1, 1M)$$

Second choice

$(.01\,, 0\,; 0.89\,, 1M\,;\ .10\,, 5M)$ $\quad$ $(1, 1M)$

The Certainty Effect

- Kahneman and Tversky wanted to “clean” Allais’s example so that it contains only two-outcome lotteries
  - And then violations of the axiom are hardly due to confusion
- Choices 4.5 and 4.10 are their example of the Certainty Effect
- The point: psychologically, 100% is more than just four times 25%

Reference

Prospect Theory: An analysis of decision under risk

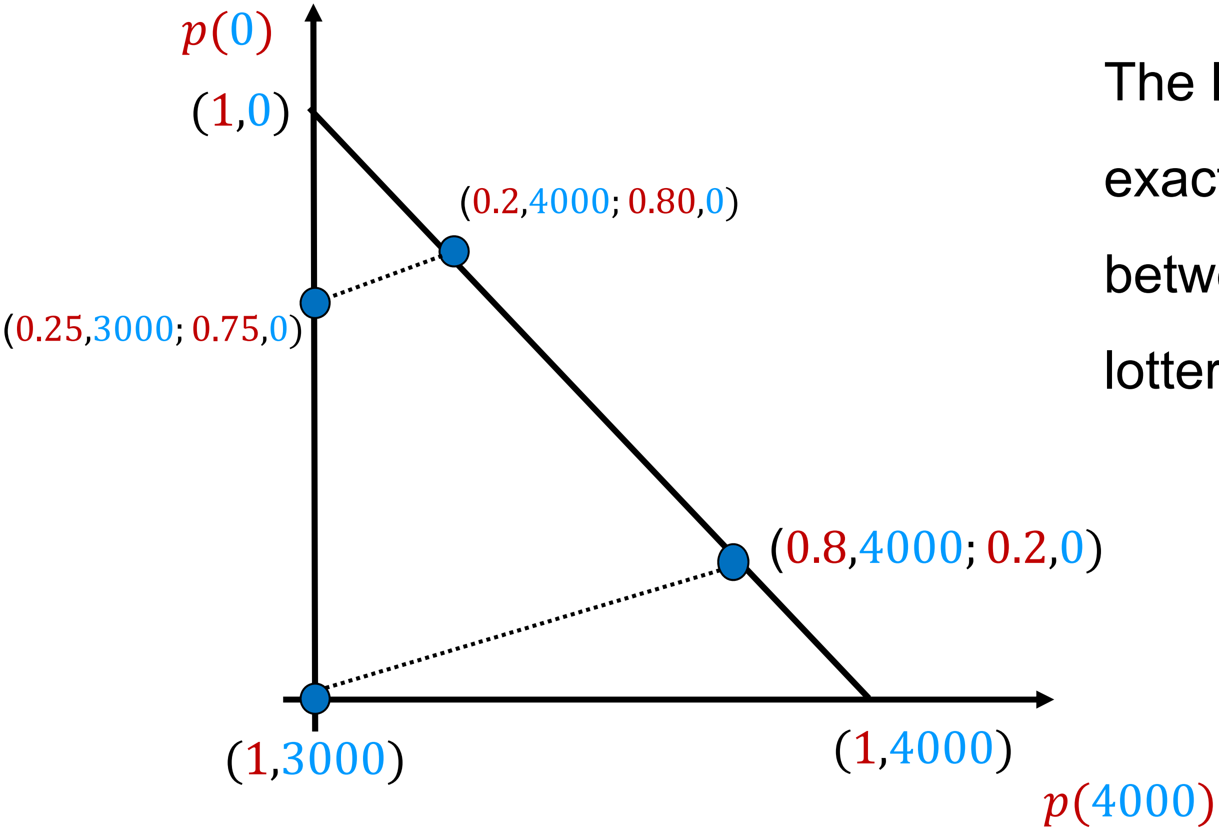
Daniel Kahneman, Amos Tversky

Econometrica, Vol. 47, No. 2 (March, 1979), pp. 263-291

Abstract

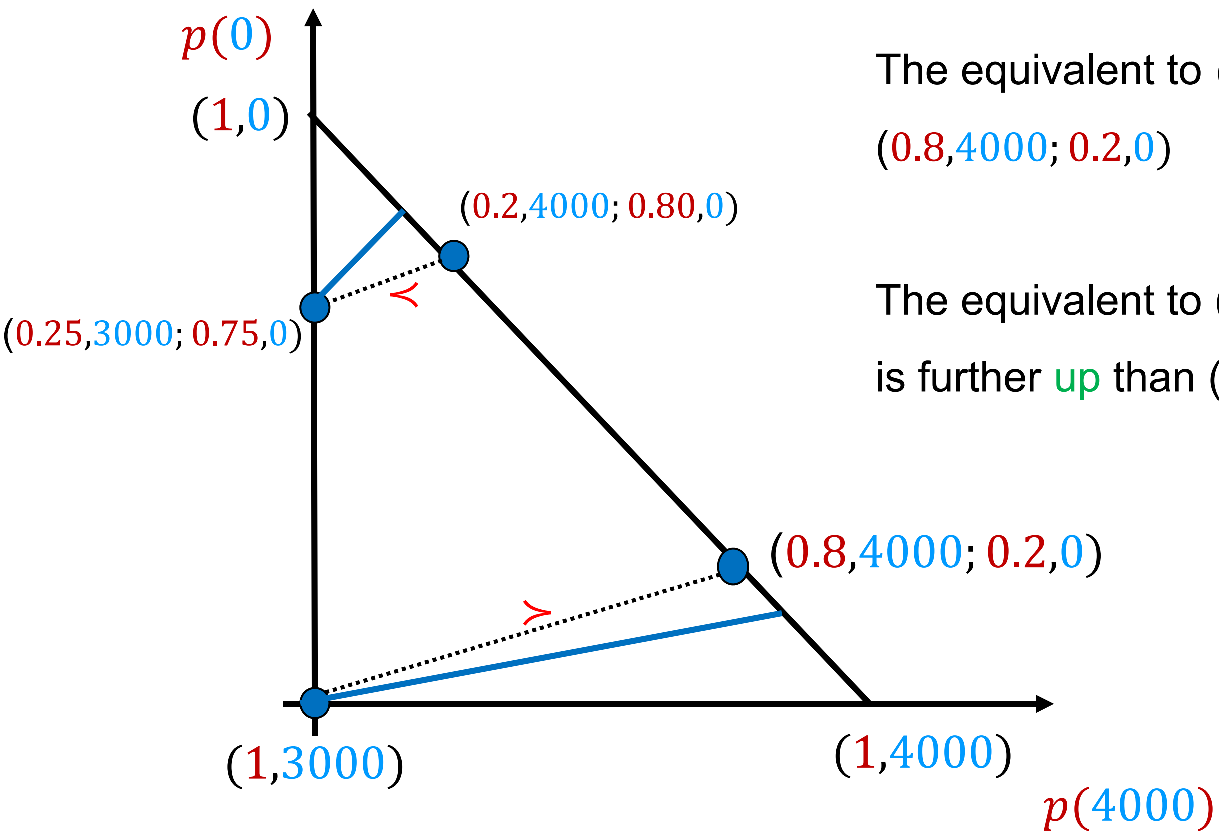
This paper presents a critique of expected utility theory as a descriptive model of decision making under risk, and develops an alternative model, called prospect theory. Choices among risky prospects exhibit several pervasive effects that are inconsistent with the basic tenets of utility theory. In particular, people underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This tendency, called the certainty effect, contributes to risk aversion in choices involving sure gains and to risk seeking in choices involving sure losses. In addition, people generally discard components that are shared by all prospects under consideration. This tendency, called the isolation effect, leads to inconsistent preferences when the same choice is presented in different forms. An alternative theory of choice is developed, in which value is assigned to gains and losses rather than to final assets and in which probabilities are replaced by decision weights. The value function is normally concave for gains, commonly convex for losses, and is generally steeper for losses than for gains. Decision weights are generally lower than the corresponding probabilities, except in the range of low probabilities. Overweighting of low probabilities may contribute to the attractiveness of both insurance and gambling.

The Certainty Effect in the Triangle



The lotteries in 4.10 are exactly 25% of the "way" between $(1,0)$ and the lotteries in 4.5

Indifference curves “fanning out”



Preferences as $\succ$ mean that

The equivalent to $(1,3000)$ is further down than $(0.8,4000;\ 0.2,0)$

The equivalent to $(0.25,3000;\ 0.75,0)$ is further up than $(0.2,4000;\ 0.8,0)$

Another example

- Do you prefer

  $\$1{,}000{,}000$ with probability 0.8

  or

  $\$2{,}000{,}000$ with probability 0.4 ?

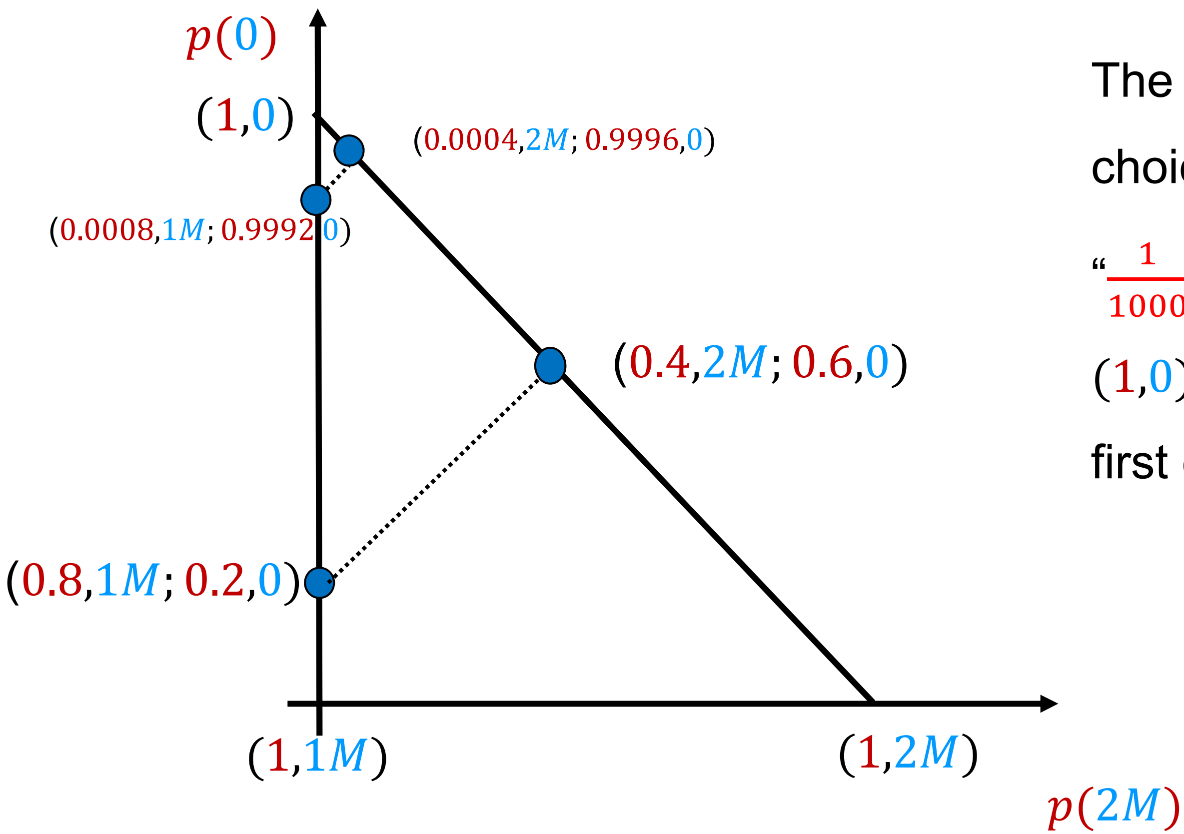
- How about

  $\$1{,}000{,}000$ with probability 0.0008

  or

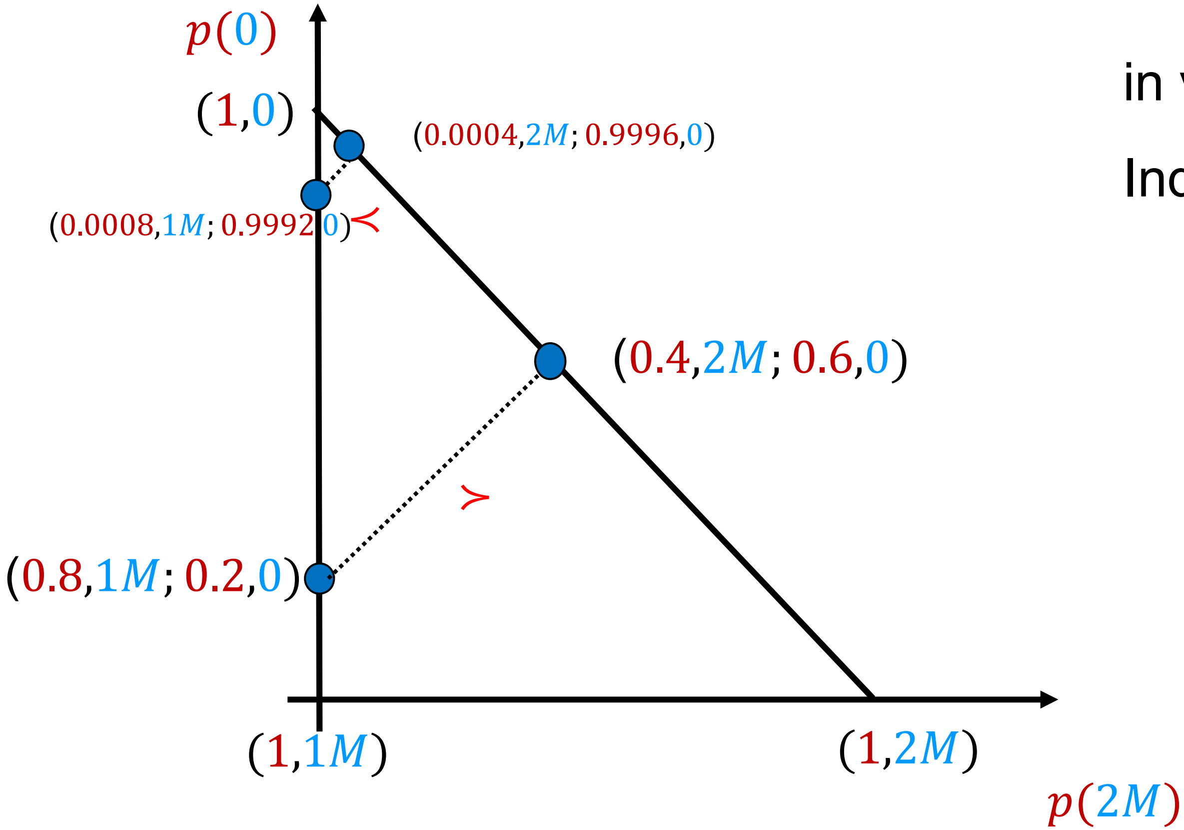
  $\$2{,}000{,}000$ with probability 0.0004 ?

Again, a Common Ratio



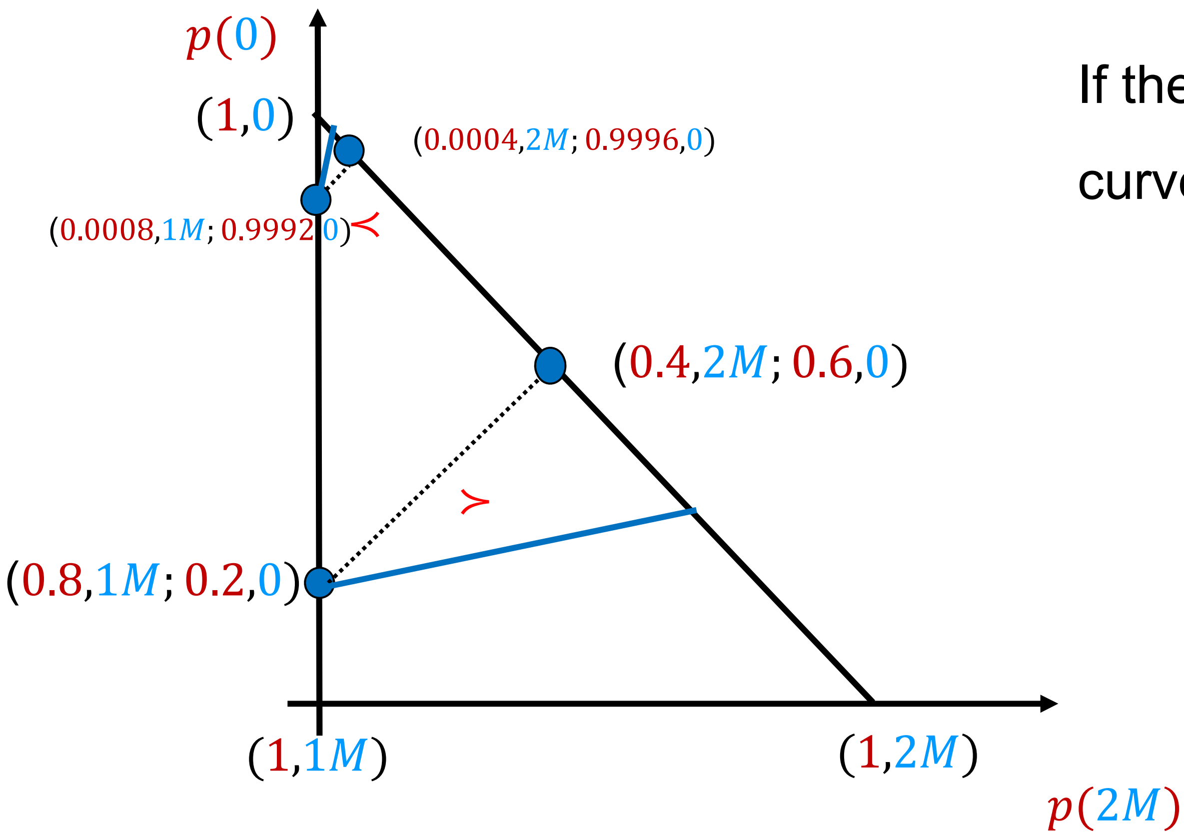
The lotteries in the second choice problem are exactly "$\frac{1}{1000}$ of the way" between $(1,0)$ and the lotteries in the first choice problem

And, again, we often observe



in violation of the Independence Axiom

Which can be explained



If the indifference curves “fan out”

A confession

$$10^{-9} = 10^{-12}$$

- I know that the first is $1{,}000$ larger than the second
- But when it comes to probabilities, I can’t tell the difference
- Can you imagine making a different decision when the stated probability is 10^{-9} vs. 10^{-12} ?

Excuses – Irrationality

$$10^{-9} = 10^{-12}$$

- Well, evolution has not prepared us to deal with these
- So we can’t really realize how small small numbers can be
- Notice that it’s much easier to tell the difference between the amounts

  $\$1,000,000$ and $\$2,000,000$

  than between the probabilities

  0.0008 and 0.0004

Rational excuses

$$10^{-9} = 10^{-12}$$

- What's the probability that whoever gave me these numbers actually knew what they were talking about?
- Shouldn't I have a healthy doubt about the reliability of these numbers?
- A similar point to some rationalizations of the compromise effect, the default effect…

Prospect Theory

In any event, Kahneman and Tversky suggested an alternative theory for decision under risk

Two main components:

- People exhibit gain-loss asymmetry
- People "distort" probabilities

Gain-Loss Asymmetry

- Positive-negative asymmetry in psychology
- A “reference point” relative to which outcomes are defined
- “Prospects” as opposed to “lotteries”
  - Same mathematical entities
  - Only the monetary values are interpreted as changes (relative to the reference point)

Zero on the utility scale

In the classical theory, we can "shift" the utility

$$v(x) = u(x) + b$$

For any b without affecting anything. For

utility maximization

expected utility maximization

discounted (expected) utility maximization…

the "zero" has no particular meaning

By contrast

Psychology suggests that there might be a "special point" on the utility/payoff scale:

Helson's adaptation level

Simon's aspiration level

Kahneman-Tversky's reference point

Adaptation Level Theory

- A theory of perception
  - The brain responds mostly to changes
  - It adapts to a certain level of the stimulus
  - The adaptation level determines what needs to be attended to

Harry Helson (1898-1977)

- Warning: we do not adapt to anything and everything

Satisficing

- A theory of decision making
  - Managers don't optimize, they satisfice
  - They put out fires
  - An aspiration level is the level of performance below which there's a problem
- Suggested (and coined the term) "Bounded Rationality"

Herbert A. Simon (1916-2001)

What determines the reference point?

Botond Kőszegi (b. 1973)

Matthew Rabin (b. 1963)

Kőszegi and Rabin: the economic environment

Reference

A model of reference-dependent preferences

Botond Kőszegi, Matthew Rabin

The Quarterly Journal of Economics, Vol. 121, No. 4 (Nov. 2006), pp. 1133-1165

Abstract

We develop a model of reference-dependent preferences and loss aversion where “gain-loss utility” is derived from standard “consumption utility” and the reference point is determined endogenously by the economic environment. We assume that a person's reference point is her rational expectations held in the recent past about outcomes, which are determined in a *personal equilibrium* by the requirement that they must be consistent with optimal behavior given expectations. In deterministic environments, choices maximize consumption utility, but gain-loss utility influences behavior when there is uncertainty. Applying the model to consumer behavior, we show that willingness to pay for a good is increasing in the expected probability of purchase and in the expected prices conditional on purchase. In within-day labor-supply decisions, a worker is less likely to continue work if income earned thus far is unexpectedly high, but more likely to show up as well as continue work if expected income is high.

Probability distortion

- Small probabilities are translated into larger "decision weights"

- As in the examples of State lotteries and insurance

- Indeed…

Small probabilities

People have a hard time understanding small probabilities

Imagine that every week a State lottery allows you to guess 6 numbers out of 47

The number of such choices is $\binom{47}{6} = 10{,}737{,}573$

And the chance of winning (not necessarily alone) in a given week is

$$\frac{1}{10{,}737{,}573} = 0.000{,}000{,}0931$$

How long should you wait to obtain a probability of winning of 1% ?

Calculation

What is n such that

$$(1 - 0.000{,}000{,}0931)^n = 0.99$$

$$n \log\ (1 - 0.000{,}000{,}0931) = \log(0.99)$$

$$n = \frac{\log\ (0.99)}{\log(0.999{,}999{,}9069)} = 107{,}916$$

Or, in years

$$\frac{107{,}916}{52} = 2{,}075$$

(And a linear approximation isn't so bad in this case, yielding

$2{,}065$ years)

Psychologists have noticed that

Roughly as soon as economists started to get excited about EU theory

- von Neumann and Morgenstern (1944,1947)
- Friedman and Savage (1948)

… there were findings that “psychological probability” isn’t “mathematical probability”

- Preston and Baratta (1948)
- Edwards (1955)

Reference

An experimental study of the auction-value of an uncertain outcome

Malcolm G. Preston, Philip Baratta

American Journal of Psychology, Vol. 61, No. 2 (April, 1948), pp. 183-193

(No abstract. An “Baratta” is the correct spelling)

Reference

The prediction of decisions among bets

Ward Edwards

Journal of Experimental Psychology, 50(3) (1955)., 201-214.

https://doi.org/10.1037/h0041692

Abstract

A very simple mathematical model was developed for predicting choices among bets. This model, based on the concepts of subjective value or utility of money and subjective probability, asserts that Ss choose the bet with the maximum subjectively expected utility. An experiment was designed to test this model… . The model predicted substantially better than chance." The efficiency of other models is discussed and "it is concluded that the subjectively expected utility maximization model is adequate to account for the results of this experiment, and that subjective probabilities are much more important than utilities in determining choices among bets such as those used in this experiment.

Decision weights

Preston and Barrata 1948

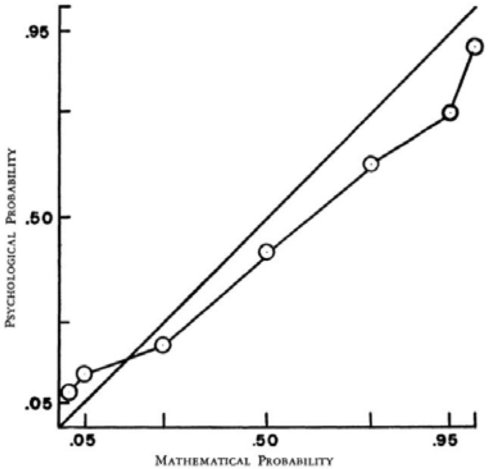


Fig. 1. Functional Relationship Between Psychological and Mathematical Probability

Kahneman and Tversky 1979

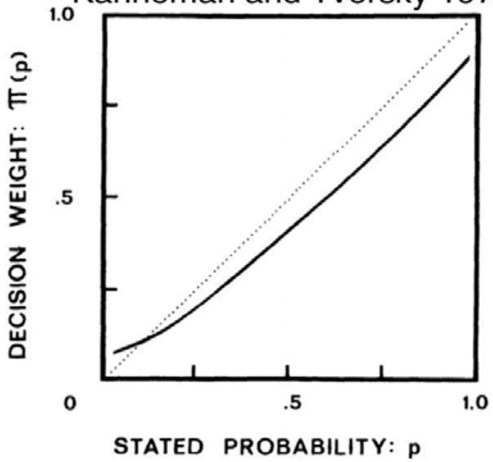


Figure 4.—A hypothetical weighting function.

How do we use decision weights?

One idea: just as D. Bernoulli suggested to switch from

(expected value) :

$$p_1 * x_1 + \cdots + p_n * x_n$$

to (expected utility)

$$p_1 * u(x_1) + \cdots + p_n * u(x_n)$$

We can now have

$$f(p_1) * u(x_1) + \cdots + f(p_n) * u(x_n)$$

Three problems

The expression

$$f(p_1) * u(x_1) + \cdots + f(p_n) * u(x_n)$$

- Depends on how we represent a lottery in case some

outcomes appear more than once

- Will be discontinuous in the outcomes (outcome utilities)

- Will behave non-monotonically in the above

First problem: definition

Suppose that $x_1 = x_2 = \$10$ and $p_1 = p_2 = 0.1$

Do we compute

$$f(0.1) * u(\$10) + f(0.1) * u(\$10) + \cdots + f(p_n) * u(x_n)$$

or

$$f(0.2) * u(\$10) + \cdots + f(p_n) * u(x_n) \quad ?$$

… unless

$$2 * f(0.1) = f(0.2)$$

this will make a difference…

More generally

If we have two probabilities p, q ($p + q \leq 1$)

such that

$$f(p+q) \neq f(p) + f(q)$$

we have a modeling decision to make

Kahneman and Tversky suggested an "editing phase" in which these probabilities are lumped together.

Second problem: continuity

Suppose that, for a small ε (positive or negative),

$$u(x_1) = 5 \qquad u(x_2) = 5 + \varepsilon$$

As long as $\varepsilon \neq 0$ we use

$$f(p_1) * 5 + f(p_2) * (5 + \varepsilon) + \cdots + f(p_n) * u(x_n) \qquad (**)$$

but when $\varepsilon = 0$ we use

$$f(p_1 + p_2) * 5 + \cdots + f(p_n) * u(x_n)$$

which isn't the limit of $(**)$

[… unless $f(p + q) = f(p) + f(q)$]

Third problem: monotonicity

Suppose that, for some p, q (with $p + q \leq 1$)

$$f(p+q) > f(p) + f(q)$$

And pick a point at which u is continuous.

(If it's monotone, most points are points of continuity, but we anyway don't want to rule out the continuous case)

Say we picked x so that

$$u(x+\varepsilon) \xrightarrow[\varepsilon \to 0]{} u(x)$$

Third problem: monotonicity

Compare $(p, x; q, x + \varepsilon \ldots ; p_n, x_n)$

with $(p, x; q, x \ldots ; p_n, x_n)$

The first is evaluated by

$$f(p) * u(x) + f(q) * u(x + \varepsilon) + \cdots + f(p_n) * u(x_n)$$

whereas the second – by

$$f(p + q) * u(x) + \cdots + f(p_n) * u(x_n)$$

But because $u(x + \varepsilon) \xrightarrow[\varepsilon \to 0]{} u(x)$

and $f(p + q) > f(p) + f(q)$

For a small $\varepsilon > 0$ we get a violation of monotonicity

And if

$$f(p+q) < f(p) + f(q)$$

With

$$u(x+\varepsilon) \xrightarrow[\varepsilon \to 0]{} u(x)$$

We again compare $(p, x; q, x+\varepsilon \ldots ; p_n, x_n)$

with $(p, x; q, x \ldots ; p_n, x_n)$

And this time for a small $\varepsilon < 0$

$$f(p) * u(x) + f(q) * u(x+\varepsilon) + \cdots + f(p_n) * u(x_n)$$

is ranked above

$$f(p+q) * u(x) + \cdots + f(p_n) * u(x_n)$$

– again a violation of monotonicity

So, to preserve monotonicity

We need to have, for all p, q (with $p + q \leq 1$)

$$f(p+q) = f(p) + f(q)$$

But, with $f(1) = 1$ this means

$$f\left(\frac{1}{2}\right) = \frac{1}{2};\ f\left(\frac{1}{n}\right) = \frac{1}{n};\ f\left(\frac{k}{n}\right) = \frac{k}{n}$$

And a monotone such f is simply the identity

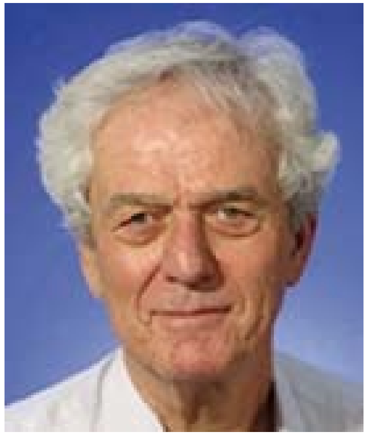
In other words, we're back to vNM's EUT

Rank Dependent Utility

Use f not for the probability to obtain an outcome, but for the probability to obtain an outcome or more

John Quiggin (b. 1956)

Chew Soo Hong (b. 1954)



Menahem Yaari (b. 1935)

The cumulative idea

Standard expected utility can be written as

$$p_1 * u(x_1) + p_2 * u(x_2) + p_3 * u(x_3)$$

or

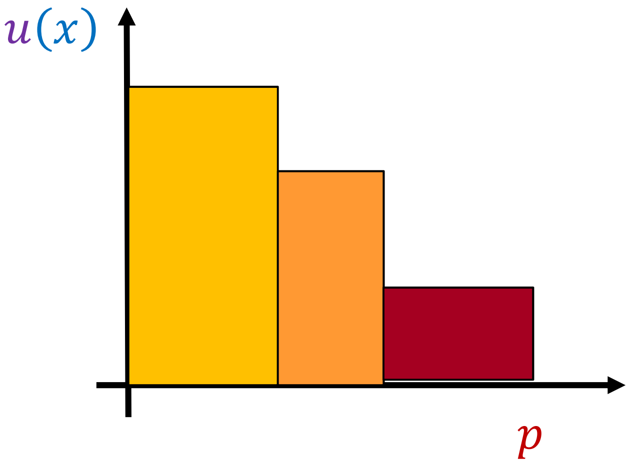
$$p_1 * [u(x_1) - u(x_2)] + (p_1 + p_2) * [u(x_2) - u(x_3)] + (p_1 + p_2 + p_3) * [u(x_3) - u(x_4)] + \ldots$$

Which is always true but makes more sense if the outcomes are ranked so that

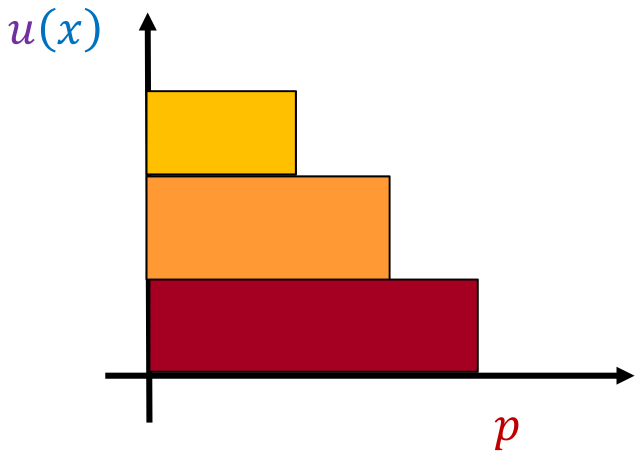
$$u(x_1) \geq u(x_2) \geq u(x_3) \geq \cdots$$

Graphically

With $u(x_1) \geq u(x_2) \geq u(x_3) \geq \cdots$



equals



$$p_1 * u(x_1) + p_2 * u(x_2) + p_3 * u(x_3)$$

$$p_1 * [u(x_1) - u(x_2)] + (p_1 + p_2) * [u(x_2) - u(x_3)] + (p_1 + p_2 + p_3) * [u(x_3) - u(x_4)] + \ldots$$

This is just re-writing

Both

$$p_1 * u(x_1) + p_2 * u(x_2) + p_3 * u(x_3)$$

$$p_1 * [u(x_1) - u(x_2)] + (p_1 + p_2) * [u(x_2) - u(x_3)] + (p_1 + p_2 + p_3) * [u(x_3) - u(x_4)] + \dots$$

Are evidently linear in probabilities

We need some non-linearity to deal with violations of Independence

But

Why introduce the non-linearity on the probability to get exactly a certain utility level as opposed to a certain level or more?

Why…

$$f(p_1) * u(x_1)$$
$$+ f(p_2) * u(x_2)$$
$$+ f(p_3) * u(x_3)$$

Rather than…

$$f(p_1) * [u(x_1) - u(x_2)]$$
$$+f(p_1 + p_2) * [u(x_2) - u(x_3)]$$
$$+f(p_1 + p_2 + p_3) * [u(x_3) - u(x_4)]$$
$$+ \dots$$

“Distorting” cumulative probabilities

Might make sense

How does the decision maker think about the lottery?

It might depend on the way it’s represented

But for many real-life applications, lotteries are not given as stated probabilities

If the lottery is a description of risk in, say, an insurance problem, the cumulative might make more sense

Notice that

We can also re-arrange the terms to look at specific outcomes again

So that

$$f(p_1) * [u(x_1) - u(x_2)]$$
$$+ f(p_1 + p_2) * [u(x_2) - u(x_3)]$$
$$+ f(p_1 + p_2 + p_3) * [u(x_3) - u(x_4)]$$
$$+ \ldots$$

can be written as

$$f(p_1) * u(x_1)$$
$$+ [f(p_1 + p_2) - f(p_1)] * u(x_2)$$
$$+ [f(p_1 + p_2 + p_3) - f(p_1 + p_2)] * u(x_3)$$
$$+ \ldots$$

… probabilities are “distorted” in a way that depends also on their rank

– hence “rank-dependent”

What did Prospect Theory say?

Kahneman and Tversky (1979) had (eq (1), (2), p. 276)

If $p + q < 1$ or $x \geq 0 \geq y$ or $x \leq 0 \leq y$ then

$$V(x, p; y, q) = \pi(p)v(x) + \pi(p)v(y)$$

which is like Edwards (1955) formula

But if $p + q = 1$ and $[x > y > 0$ or $x < y < 0]$ then

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)]$$

which is like the rank-dependent one

– and then something similar to the former in the appendix

Cumulative Prospect Theory

Tversky and Kahneman (1992) adopted the rank-dependent idea and coupled it with the notion of a reference points

Though there are debates

See Bernheim and Sprenger (2020)

Reference

A theory of anticipated utility

John Quiggin

Journal of Economic Behavior and Organization, Vol. 3, No. 4 (Dec., 1982), pp. 323-343

Abstract

A new theory of cardinal utility, with an associated set of axioms, is presented. It is a generalization of the von Neumann-Morgenstern expected utility theory, which permits the analysis of phenomena associated with the distortion of subjective probability.

Reference

A generalization of the quasilinear mean with applications to the measurement of income inequality and decision theory resolving the Allais Paradox

Chew Soo Hong

Econometrica, Vol. 51, No. 4 (July, 1983), pp. 1065-1092

Abstract

The main result of this paper is a generalization of the quasilinear mean of Nagumo [29], Kolmogorov [26], and de Finetti [17]. We prove that the most general class of mean values, denoted by M, satisfying Consistency with Certainty, Betweenness, Substitution-independence, Continuity, and Extension, is characterized by a continuous, nonvanishing weight function a and a continuous, strictly monotone value-like function. The quasilinear mean M results whenever the weight function is constant. Existence conditions and consistency conditions with first and higher degree stochastic dominance are derived and an extension of a well known inequality among quasilinear means, which is related to Pratt's [31] condition for comparative risk aversion, is obtained. Under the interpretation of mean value as a certainty equivalent for a lottery, the M mean gives rise to a generalization of the expected utility hypothesis which has testable implications, one of which is the resolution of the Allias "paradox." The M mean can also be used to model the equally-distributed-equivalent or representative income corresponding to an income distribution. This generates a family of relative and absolute inequality measures and a related family of weighted utilitarian social welfare functions.

Reference

The dual theory of choice under risk

Menahem Yaari

Econometrica, Vol. 55, No. 1 (Jan., 1987), pp. 95-115

Abstract

This paper investigates the consequences of the following modification of expected utility theory: Instead of requiring independence with respect to probability mixtures of risky prospects, require independence with respect to direct mixing of payments of risky prospects. A new theory of choice under risk-a so-called dual theory-is obtained. Within this new theory, the following questions are considered: (i) numerical representation of preferences; (ii) properties of the utility function; (iii) the possibility for resolving the "paradoxes" of expected utility theory; (iv) the characterization of risk aversion; (v) comparative statics. The paper ends with a discussion of other non-expected-utility theories proposed recently.

Reference

Advances in Prospect Theory: Cumulative representation of uncertainty

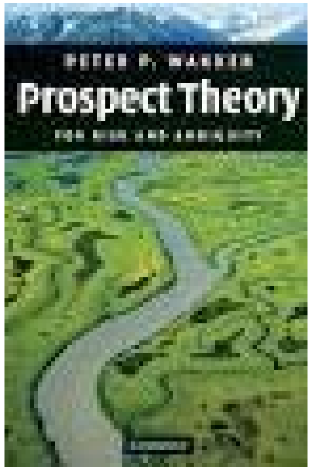
Amos Tversky, Daniel Kahneman

Journal of Risk and Uncertainty, Vol. 5 (1992), pp. 297-323

Abstract

We develop a new version of prospect theory that employs cumulative rather than separable decision weights and extends the theory in several respects. This version, called cumulative prospect theory, applies to uncertain as well as to risky prospects with any number of outcomes, and it allows different weighting functionsfor gains and for losses. Two principles, diminishing sensitivity and loss aversion, are invoked to explain the characteristic curvature of the value function and the weighting functions. A review of the experimental evidence and the results of a new experiment confirm a distinctive fourfold pattern of risk attitudes: risk aversion for gains and risk seeking for losses of high probability; risk seeking for gains and risk aversion for losses of low probability.

A great textbook



Peter P. Wakker (b. 1956)

Prospect Theory: For risk and ambiguity

Cambridge University Press, 2010

Critique I

Direct tests of Cumulative Prospect Theory

B. Douglas Bernheim, Charles Sprenger

Econometrica, Vol. XX (2020), pp. xx-yy

Abstract

Cumulative Prospect Theory (CPT), the leading behavioral account of decision making under uncertainty, assumes that the probability weight applied to a given outcome depends on its ranking. This assumption is needed to avoid the violations of dominance implied by Prospect Theory (PT). We devise a simple and direct non-parametric method for measuring the change in relative probability weights resulting from a change in payoff ranks. We find no evidence that these weights are even modestly sensitive to ranks. The estimated changes in relative weights range from +3% to -3%, and in no case can we reject the hypothesis of rank-independence. Our estimates rule out changes in relative probability weights larger than a few percent as ranks change with 95% confidence. In contrast, conventional calibrations of CPT preferences for the same subjects imply that probability weights should change by 20% to 40%. Models with reference distributions (notably Koszegi and Rabin, 2006) have similar implications, and hence we falsify them as well. Additional tests nevertheless indicate that the dominance patterns predicted by PT do not arise. We reconcile these findings by positing a form of complexity aversion that generalizes the well-known certainty effect.

Critique II

Decisions from experience and the effect of rare events in risky choice

Ralph Hertwig, Greg Barron, Elke U. Weber, Ido Erev

Psychological Science, Vol. 15 No. 8 (2004), pp. 534-539

Abstract

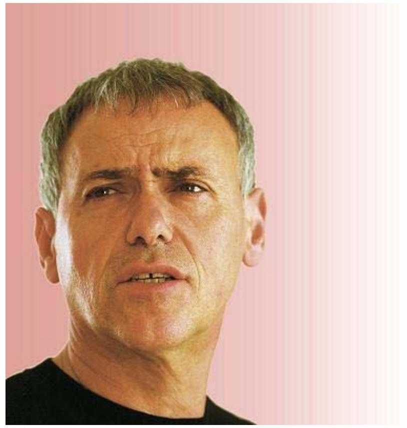
When people have access to information sources such as newspaper weather forecasts, drug-package inserts, and mutual-fund brochures, all of which provide convenient descriptions of risky prospects, they can make decisions from description. When people must decide whether to back up their computer's hard drive, cross a busy street, or go out on a date, however, they typically do not have any summary description of the possible outcomes or their likelihoods. For such decisions, people can call only on their own encounters with such prospects, making decisions from experience. Decisions from experience and decisions from description can lead to dramatically different choice behavior. In the case of decisions from description, people make choices as if they overweight the probability of rare events, as described by prospect theory. We found that in the case of decisions from experience, in contrast, people …

Rubinstein's critique

- It's not about playing with the formulae
- We need to look at the mental process people go through
- In the Certainty Effect, comparing

$$(4{,}000, 0.20) \text{ with } (3{,}000, 0.25)$$

people ignore the probabilities because they are similar



Ariel Rubinstein (b. 1951)

Reference

Similarity and decision-making under risk (is there a utility theory resolution to the Allais paradox?)

Ariel Rubinstein

Journal of Economic Theory, Vol. 46 No.1 (1988), pp. 145-153

Abstract

It is argued that the Allais paradox reveals a certain property of decision scheme we use to determine the preference of one lottery over another. The decision scheme is based on the use of similarity relations on the probability and prize spaces.

It is proved that for every pair of similarity relations there is essentially only one preference consistent with the decision scheme and the similarities. It is claimed that the result shows a basic difficulty in reconciling utility theory with experimental data.

DECISION UNDER

UNCERTAINTY

Subjective Expected Utility

Uncertainty

- As mentioned above, Pascal (1670) already used (subjective) probabilities to discuss the problem of becoming a believer
- Bayes (1763) used subjective probabilities to convince us that God exists
- Can we always quantify uncertainty probabilistically?

Answer 1: No, we cannot

Knight distinguished between these two types of uncertainty

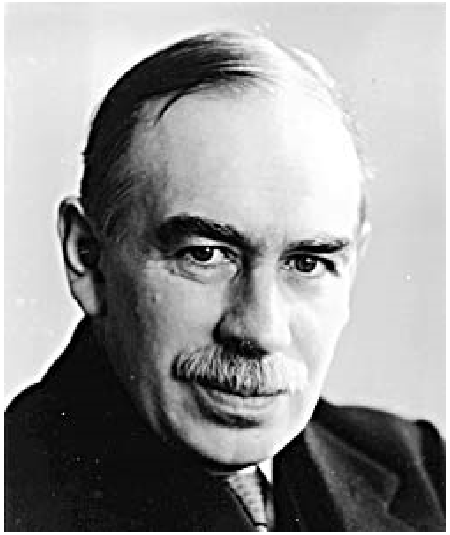
"Knightian uncertainty" – cannot be quantified

Argued that entrepreneurs are more tolerant of that type uncertainty

Frank Knight (1885-1972)

And this was also Keynes's view

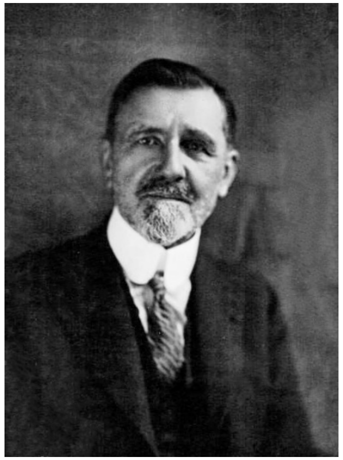
"...About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know."



John Maynard Keynes (1883-1946)

On the other hand…

There were others…



Émile Borel (1871-1956)

Frank P. Ramsey (1903-1930)

Bruno de Finetti (1906-1985)

Subjective probabilities and choice

- Borel, Ramsey, de Finetti: subjective probability should be measured by betting behavior

  "Put your money where your mouth is"

- Coherent betting behavior ⇒ maximization of expectation

- Define probabilities by behavior

Savage's result

Leonard J. Savage
(1917-1971)

- An amazing theorem that shows
- Coherent decisions ⇒ expected utility maximization with respect to a subjective probability
- Both the probability and the utility are derived from preferences
- It convinced the entire field. Make it "fields".

Savage's model

S – states

X – outcomes

Acts

$$F = X^S = \{ f \mid f : S \to X \}$$

Events are subsets of S

What you see is what you get

measurable?

S – states

Topological ?

X – outcomes linear?

Acts

$$F = X^S = \{ f \mid f : S \to X \}$$

Events are subsets of S

measurable?

Savage's model

We will assume as observable $\succcurlyeq$ on F

From which we can also derive $\succcurlyeq$ on X

And we can compare $f \in F$ with $x \in X$

For $f, g \in F$ and A, define $fAg \in F$

$$(fAg)(s) = \begin{Bmatrix} g(s) & s \in A \\ f(s) & s \in A^c \end{Bmatrix}$$

Savage's axioms

P1. $\succcurlyeq$ on F is complete and transitive

P2. $hAf \succcurlyeq hAg \Leftrightarrow h'Af \succcurlyeq h'Ag$ $\quad (f \succcurlyeq_A g)$

P3. $hAx \succcurlyeq hAy \Leftrightarrow x \succcurlyeq y$ $\quad$ provided that A is non-null ($\exists f \succ_A g$)

P4. $xAy \succcurlyeq xBy \Leftrightarrow zAw \succcurlyeq zBw$ $\quad$ provided that $x \succ y, z \succ w$

P5. There exist $f \succ g$

P6. If $f \succ g$, then for any h there exists a partition $\{A_1, \dots, A_n\}$ of S

such that $fA_ih \succ g$ and $f \succ gA_ih$ for all i

P7. $f \succcurlyeq_A g(s)$ for all $s \in A \Rightarrow f \succcurlyeq_A g$

$f(s) \succcurlyeq_A g$ for all $s \in A \Rightarrow f \succcurlyeq_A g$

Savage's P1

P1.$\succcurlyeq$ on F is complete and transitive

Transitivity is rather standard

Completeness is a bigger issue than it is in consumer theory

And will become even bigger an issue when the state space gets expanded

Savage's P2

P2. $hAf \succcurlyeq hAg \Leftrightarrow h'Af \succcurlyeq h'Ag \quad (f \succcurlyeq_A g)$

The Sure Thing Principle: if A doesn't happen, we get the same outcome

Who cares which same outcome (h or h')?

Savage's P2 – an example

P2. $hAf \succcurlyeq hAg \Leftrightarrow h'Af \succcurlyeq h'Ag \quad (f \succcurlyeq_A g)$

	A^c		A	
States/ acts	1	2	3	4
hAf	10	7	10	0
hAg	10	7	5	5
$h'Af$	3	8	10	0
$h'Ag$	3	8	5	5

The question should be, do you prefer f to g given A

Why “The Sure Thing Principle”?

Imagine that f is preferred to g given A and given A^c

	A^c		A	
States/ acts	1	2	3	4
f	10	7	10	0
g	5	12	5	5
h	10	7	5	5

Introduce h that equals f on A^c and equals g on A

$f \succcurlyeq h$ ($f \succcurlyeq_A g$)

$h \succcurlyeq g$ ($f \succcurlyeq_{A^c} g$) … hence $f \succcurlyeq g$

Savage's P3

P3. $hAx \succcurlyeq hAy \Leftrightarrow x \succcurlyeq y$ provided that A is non-null ($\exists f \succ_A g$)

The most natural way to state monotonicity

As we shall see, a bit more is assumed

But – can you state monotonicity otherwise (without any structure on outcomes)?

Notice that we want A to be non-null for $x \succ y \Rightarrow hAx \succ hAy$

Savage's P4

P4. $xAy \succcurlyeq xBy \Leftrightarrow zAw \succcurlyeq zBw$ provided that $x \succ y, z \succ w$

The basic idea of Borel, Ramsey, de Finetti: measure subjective likelihood by the willingness to bet

But what if the willingness to bet on $\$100$ gives different results than on $\$200$?

P4 demands that this not be the case

Savage's P5

P5. There exist $f \succ g$

Is this an axiom? Is it worth mentioning?

After all, if P5 doesn't hold EU representation surely exists (for any constant u)

But it is an important reminder of the nature of the exercise: gleaning beliefs from preferences. It won't work if someone doesn't care about anything (and we won't get uniqueness of the probability)

Savage's P6

P6. If $f \succ g$, then for any h there exists a partition $\{A_1, \ldots, A_n\}$ of S such that $fA_ih \succ g$ and $f \succ gA_ih$ for all i

OK, we knew that some "technical" axiom would be needed

"technical" – about continuity/Archimedianity; also not really testable

It has a strong flavor of continuity (changing one of the acts on one "small" event doesn't change strict preference)

But P6 does more

P6. $f \succ g$, for any h there exists a partition $\{A_1, \dots, A_n\}$ of S such that

$fA_ih \succ g$ and $f \succ gA_ih$ for all i

It also guarantees "non-atomicity"

Every event with a positive weight can be further split into sub-events none of which is null

There are several notions of "non-atomicity", and P6 guarantees the strongest

P6 implies that S is infinite

P6. $f \succ g$, for any h there exists a partition $\{A_1, \dots, A_n\}$ of S such that

$fA_ih \succ g$ and $f \succ gA_ih$ for all i

Just to get the flavor: if $S = \{1, \dots, n\}$ let

$$f = (1,0,0,0, \dots, 0)$$

$$g = (0,0,0,0, \dots, 0)$$

$$h = (0,0,0,0, \dots, 0)$$

Consider any partition $\{A_1, \dots, A_n\}$ and notice that state 1 has to be in one of the A_i's. Then $fA_ih = g$.

Savage's P7

P7. $f \succcurlyeq_A g(s)$ for all $s \in A \Rightarrow f \succcurlyeq_A g$

$f(s) \succcurlyeq_A g$ for all $s \in A \Rightarrow f \succcurlyeq_A g$

It's very surprising that this is needed at all

Especially that without it we get the representation for a finite set of outcomes

(And with it we can do without P3)

An interesting comment

Savage's P3 is Redundant

Lorenz Hartmann

Econometrica, Vol. 88 No. 1 (Jan. 2020), pp. 203-205

Abstract

Savage (1954) provided the first axiomatic characterization of expected utility without relying on any given probabilities or utilities. It is the most famous preference axiomatization existing. This note shows that Savage's axiom P3 is implied by the other axioms, which reveals its redundancy. It is remarkable that this was not noticed before as Savage's axiomatization has been studied and taught by hundreds of researchers for more than six decades.

Savage's Theorem

$\succcurlyeq$ on F satisfies P1-P7

IFF

There exist a bounded and non-constant $u: X \to R$ and a finitely additive non-atomic measure p on S such that, for every f, g

$$f \succcurlyeq g \Leftrightarrow \int u\big(f(s)\big)dp \geq \int u\big(g(s)\big)dp$$

Savage's Theorem – uniqueness

$\succcurlyeq$ on F satisfies P1-P7

IFF

There exist a bounded and non-constant $u: X \to R$

and a finitely additive non-atomic measure p on S

such that, for every f, g

$$f \succcurlyeq g \Leftrightarrow \int u(f(s))dp \geq \int u(g(s))dp$$

In this case u is unique up to a positive affine transformation and p is unique

What is “finitely additive”?

A probability measure p should satisfy

$$p(A \cup B) = p(A) + p(B)$$

whenever the events A, B are disjoint

Which also means that for every pairwise disjoint

$\{A_i\}_{i=1,\dots,n}$ we have

$$p\left(\cup_{i=1,\dots,n} A_i\right) = \sum_{i=1}^{n} p(A_i)$$

Countable additivity

(or σ-additivity) also means that we can take the limits

So that for every pairwise disjoint $\{A_i\}_{i=1,..,\infty}$ we also have

$$p\left(\cup_{i=1,..,\infty} A_i\right) = \sum_{i=1}^{\infty} p(A_i)$$

A big conceptual debate behind this assumption

Countable additivity

often comes at the cost of declaring many events "unmeasurable"

This looks rather artificial if one starts with a state space such as $[0,1]$

But less so if the state space is generated as truth values of propositions

The syntactic approach

We start with propositions that can be true or false

Say, there are only countably many of these $p_1, p_2, p_3, \ldots$

(consider that which can be expressed in finite-length sentences)

A state s should assign to each p_i a truth value – a number in $\{0,1\}$

Thus

A number in $x \in [0,1]$ that can be written in a binary expansion

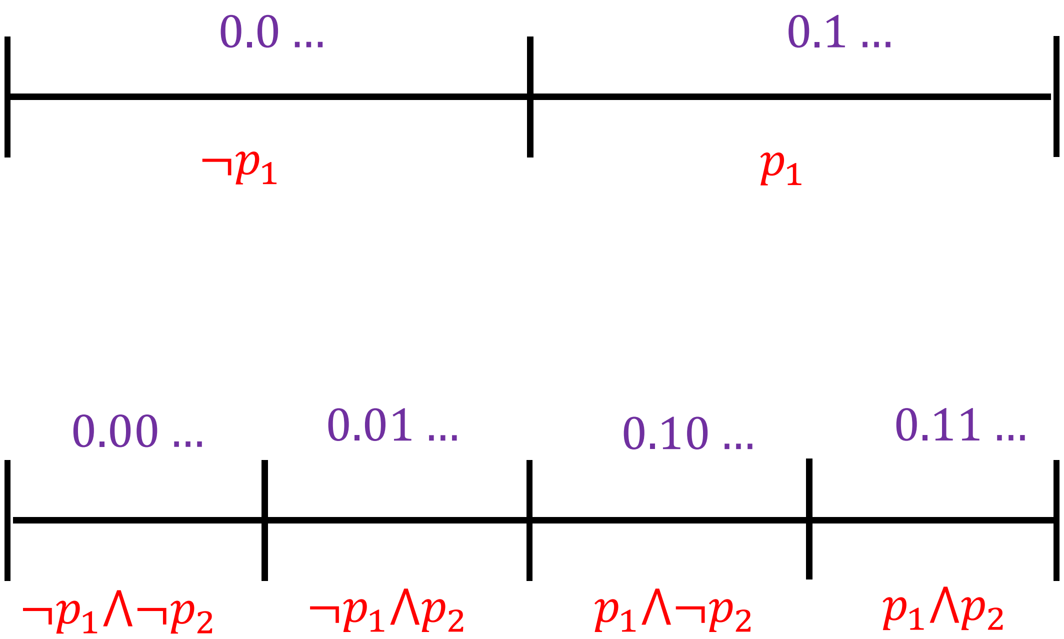
$$x = 0.1001101 \ldots$$

Can be read as providing the truth value for each of the propositions $p_1, p_2, p_3, \ldots$

(according to the agreed-upon enumeration)

It should better be logically consistent

Binary expansions



A problem

… the same x can have more than one expansion

$$x = 0.1000\ldots = 0.0111\ldots$$

So we can use 2 rather than 1 (and use only numbers in the Cantor set) or whatever

In any event

We can think of the state space as derived from more primitive propositions

And then the restriction to measurable sets isn't so artificial

In fact, the Borel algebra corresponds to events that can be stated in finite sentences in a natural language

What is “non-atomic”?

There are several definitions that coincide for σ-additive measure but not for finitely additive ones

We use (and Savage gets) the strongest:

For any event A and every $r \in [0,1]$ there is a $B \subseteq A$, such that

$$p(B) = rp(A)$$

Aumann's critique of P3

x – swimsuit

y – umbrella

First,

P3. $hAx \succcurlyeq hAy \Leftrightarrow x \succcurlyeq y$

$(hBx \succcurlyeq hBy \Leftrightarrow x \succcurlyeq y)$

might not make sense if A is "rain" and B is "sun"

Aumann's critique of P4

x – swimsuit

y – umbrella

Next,

P4. $xAy \succcurlyeq xBy \Leftrightarrow zAw \succcurlyeq zBw$

Say, A is "rain" and $B = A^c$ is "sun"

If I'm a wimp who prefers $umbrella$ to a $swimsuit$ you might infer that I think that $Rain$ is more likely than Sun

Which might be true but doesn't follow from the ranking

$$(Rain, Sun)$$

$$(\text{umbrella}, \text{swimsuit})$$
$$\curlyvee$$
$$(\text{umbrella}, \text{umbrella})$$
$$\curlyvee$$
$$(\text{swimsuit}, \text{swimsuit})$$
$$\curlyvee$$
$$(\text{swimsuit}, \text{umbrella})$$

State dependent utility

We could think of

$$u : X \times S \to R$$

such that, for every f, g

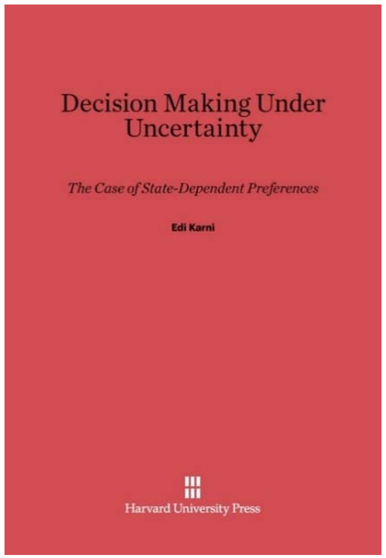
$$f \succcurlyeq g \quad \Leftrightarrow \quad \int u(f(s), s)dp \geq \int u(g(s), s)dp$$

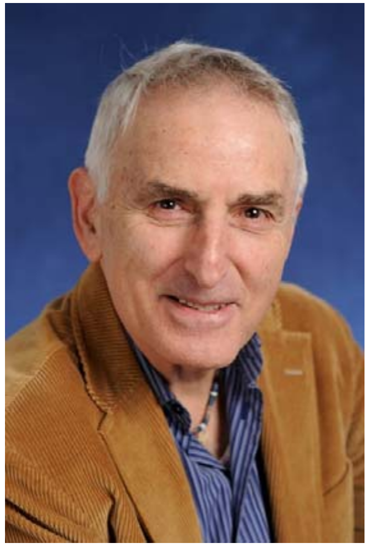
A major conceptual cost: we would lose uniqueness of p

Only the product $u(x, s)p(s)$ is observable

Proponents of state dependent utility

Jacques Drèze (b. 1929)





Edi Karni (b. 1944)

An interesting example

Aumann: suppose that one's spouse has to undergo a serious surgery. The states:

$$S = \{death, life\}$$

The known probabilities

$$q(death) = q(life) = 0.5$$

In case the spouse dies, one can't enjoy life as much

For any x

$$u\,(x, death) = 0.5 * u(x, life)$$

The example – cont.

The decision maker will satisfy all of Savage's axioms, but the probability Savage derives won't reflect the beliefs

For example, suppose that the utility from money is linear:

$$(300,0) \sim (100,100)$$

Because

$$0.5 * 0.5 * 300 + 0.5 * 0 = 75 = 0.5 * 0.5 * 100 + 0.5 * 100$$

Compatible with state-independent utility and beliefs

$$p(death) = \frac{1}{3}, p(life) = \frac{2}{3}$$

The example – cont.

- The example is a bit knife-edged
- If we add other decisions (what to tell the children?) we'll get violations of P4
- Still, it raises an important question regarding the validity of choice data as a way to measure beliefs

Reference

On the determination of subjective probability by choice

Edi Karni, Philippe Mongin

Management Science, Vol. 46, No. 2 (Feb. 2000), pp. 233-248

Abstract

The paper explores the uniqueness properties of the subjective probabilities in two axiomatizations of state-dependent preferences. Karni, Schmeidler, and Vind's (KSV 1983) system depends on selecting an arbitrary auxiliary probability, and as such, does not guarantee the uniqueness of the derived subjective probability. However, an axiom system initially designed by Karni and Schmeidler (KS 1981) and further elaborated upon here does guarantee the desired uniqueness as well as a useful property of "stability" of the derived solution. When the preference relation displays state-independence, even the KS probabilities may not agree with those derived from the classic Anscombe-Aumann (AA 1963) theorem. However, we claim that, in this case, the KS rather than the AA probabilities are the appropriate representation of the agent's beliefs.

Savage's response

These

x – swimsuit

y – umbrella

aren't outcomes

An "outcome" should specify all that matters to your well-being, such as

"lying on the beach in the sun wearing a swimsuit"

or "running for shelter half naked in the rain"

A typical theorist's response

In many cases of counter-examples to a theory we tend to go back and re-define the terms

In picking the second-least-expensive wine we redefined the "outcome"

In dynamically inconsistent choice, we redefined the "agent"

– often we use the theory as a conceptual framework

Is this response dishonest?

Well, not necessarily

It gives theory a lot of power in terms of

understanding, seeing analogies, criticizing logic…

But it may not be possible when looking at data

And it can be ridiculous…

And Savage admitted…

… that the re-definition doesn't always work

"I should not mind being hung so long as it be done without damage to my health or reputation"

When is state dependence essential?

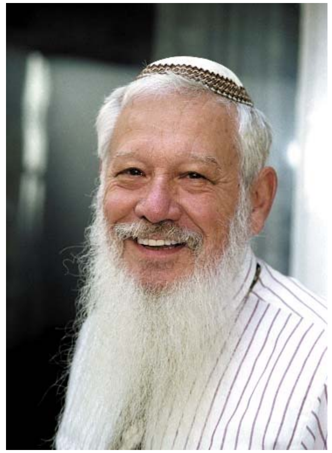
Claim: in problems involving our ability to enjoy life

(Problems of life and death, physical and mental health…)

Reason: the entire project can be viewed as measuring belief using preferences. Our "hedonic engine" is part of the measurement device. Uncertainty about the measurement device is a problem.

Anscombe and Aumann

Francis J. Anscombe (1918-2001)



Robert J. Aumann (b. 1930)

A result that is conceptually similar to Savage,

deriving subjective from objective probabilities

Anscombe-Aumann's model

S – states

X – outcomes

L – lotteries over outcomes (with finite support)

Acts

$$F \subseteq L^S = \{ f \mid f : S \to L \}$$

Events are subsets of S

But if S is infinite we assume a measurable space and limit attention to measurable subsets and simple measurable acts (assuming finitely many values)

Anscombe-Aumann's model

We will assume as observable $\succcurlyeq$ on F

From which we can also derive $\succcurlyeq$ on L

And we can compare $f \in F$ with $P \in L$

For $f, g \in F$ and A, and $\alpha \in [0,1]$ define

$$\alpha f + (1 - \alpha) g \in F$$

pointwise (state-by-state)

The mixture operation – example

x	0.4	0.8
x'	0.6	0.2
	s	s'

f

x	1	0
x'	0	1
	s	s'

g

x	0.7	0.4
x'	0.3	0.6
	s	s'

$0.5f + 0.5g$

Anscombe-Aumann's axioms

AA1. $\succcurlyeq$ on F is complete and transitive

AA2. $f \succ g \succ h \Rightarrow \exists\ \alpha, \beta \in (0,1)$ such that

$$\alpha f + (1-\alpha)h \succ g \succ \beta f + (1-\beta)h$$

AA3. $f \succ g \Rightarrow \forall\ h, \alpha > 0$

$$\alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$$

AA4. $[\forall\ s \quad f(s) \succcurlyeq g(s)] \Rightarrow f \succcurlyeq g$

AA5. There exist $f \succ g$

Axiom AA1

AA1. $\succcurlyeq$ on F is complete and transitive

Weak order – rather standard by now, with the recognition that completeness isn't so trivial when thinking about acts, and may get particularly thorny if the state space gets wild

Axiom AA2

AA2. $f \succ g \succ h \Rightarrow \exists\ \alpha, \beta \in (0,1)$ such that

$$\alpha f + (1 - \alpha) h \succ g \succ \beta f + (1 - \beta) h$$

Continuity – as in vNM's theorem

We shouldn't make a fuss here, especially that it's not really refutable

Axiom AA3

AA3. $f \succ g \Rightarrow \forall\, h, \alpha > 0$

$$\alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$$

Independence – as in vNM's theorem, but here over vectors of lotteries, not only over lotteries

When applied to constant acts (which are like vNM lotteries) – we have all the known problems of EUT under risk

But here we can also have additional issues

Axiom AA4

AA4. $[\forall\, s \quad f(s) \succcurlyeq g(s)] \Rightarrow f \succcurlyeq g$

Monotonicity – sounds innocuous, but, as Aumann pointed out to Savage (though a few years later), it sneaks in state-independence of the utility function

Axiom AA5

AA5. There exist $f \succ g$

Non-triviality – as in Savage's model

Anscombe-Aumann's Theorem

$\succcurlyeq$ on F satisfies AA1-AA5

IFF

There exist a non-constant $u: X \to R$

and a measure p on S such that, for every f, g

$$f \succcurlyeq g \Leftrightarrow \int Eu(f(s))dp \geq \int Eu(g(s))dp$$

Anscombe-Aumann's Theorem – uniqueness

$\succcurlyeq$ on F satisfies AA1-AA5

IFF

There exist a non-constant $u: X \rightarrow R$

and a measure p on S such that, for every f, g

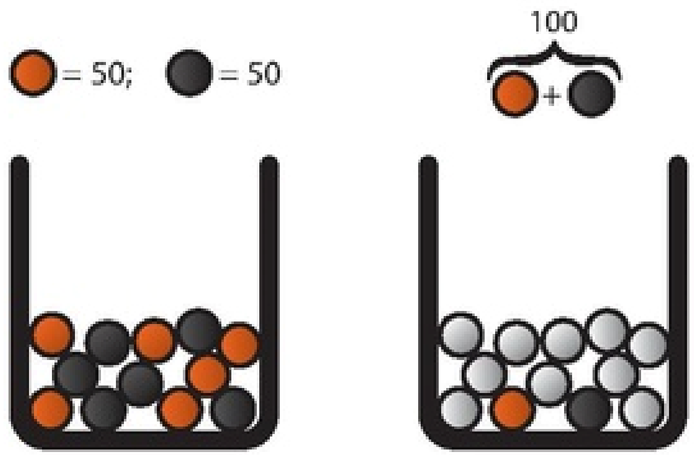
$$f \succcurlyeq g \Leftrightarrow \int Eu\big(f(s)\big)dp \geq \int Eu\big(g(s)\big)dp$$

In this case u is unique up to a positive affine transformation and p is unique

Difficulties and

Alternative Theories

Ellsberg's Paradox(es)



Daniel Ellsberg
(b. 1931)

Would you prefer to bet on the known or unknown?

Ellsberg's Two-Urn Paradox

- If you're not indifferent, you're probably not Bayesian
- A Bayesian would have to have probabilities that add up to 1
- In case of symmetry, 50%-50%
- There no distinction between 50%-50% that results from statistics and 50%-50% that comes with a shrug of shoulders
- Keynes (1921) gave a similar example

Reference

Risk, ambiguity, and the Savage axioms

Daniel Ellsberg

The Quarterly Journal of Economics, Vol. 75, No. 4 (Nov., 1961), pp. 643-669

Abstract

I. Are there uncertainties that are not risks?

II. Uncertainties that are not risks

III. Why are some uncertainties not risks?

Ellsberg's Paradox and the Independence Axiom

Let's focus on the "unknown" urn

There be two states $S = \{R, B\}$

Betting on Red gives the vector of winning probabilities $(1,0)$

Betting on Black gives the vector of winning probabilities $(0,1)$

They are equivalent

Independence means that they should be equivalent also to

$$\frac{1}{2}(1,0) + \frac{1}{2}(0,1) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

– which is what the known urn yields

In more detail

100	1	0
0	0	1
	Red	$Black$

Red

100	0	1
0	1	0
	Red	$Black$

$Black$

100	0.5	0.5
0	0.5	0.5
	Red	$Black$

$0.5Red + 0.5Black$

Ellsberg's Single-Urn Paradox

There is an urn containing 90 balls. Each ball can be red, blue, or yellow. You are also told that there are precisely 30 red balls in the urn. Hence, the remaining 60 balls are blue or yellow, but you don't know how many are blue and how many are yellow.

A ball is to be drawn at random from the urn. You are offered choices between pairs of bets, where "betting on an event" implies winning $\$100$ if the event occurs, and nothing otherwise:

Betting on the ball being red

vs.

betting on the ball being blue

Betting on the ball being not red

vs.

betting on the ball being not blue

Ellsberg's Single-Urn Paradox

- Many prefer betting on red to betting on blue (or yellow)
- But also betting on not-red to betting on not-blue (or not-yellow)

- In both cases, the bets defined by red have known probabilities, as opposed to those defined by blue (or yellow)

Is this rational?

	Red	Blue	Yellow
Red	100	0	0
Blue	0	100	0
Not red	0	100	100
Not blue	100	0	100

Focus on Red and Blue

	Red	Blue	Yellow
Red	100	0	0
Blue	0	100	0
Not red	0	100	100
Not blue	100	0	100

Why should preferences depend on the outcome for Yellow?

	Red	Blue	Yellow
Red	100	0	0
Blue	0	100	0
Not red	0	100	100
Not blue	100	0	100

Let us change the outcome for Yellow

	Red	Blue	Yellow
Red	100	0	100
Blue	0	100	100
Not red	0	100	100
Not blue	100	0	100

The Sure Thing Principle

is directly violated by these preferences in the Single-Urn experiment

The reason could be that changing the two acts, where they are equal, while keeping them equal to each other, has an asymmetric effect on their ambiguity

And act whose distribution was unambiguous (with known distribution) became ambiguous and the other – vice versa

Back to the two urns

In order to see that the Sure Thing Principle is violated in this problem, we first have to define the state space

Recall that states assign outcomes to acts

From each urn we may draw either a red or a black ball

Hence there are (at least) four states of the world:

The states of the world

	Ball drawn out of Urn A	Ball drawn out of Urn B
State 1	red	red
State 2	red	black
State 3	black	red
State 4	black	black

The decision matrix

	State 1 (RR)	State 2 (RB)	State 3 (BR)	State 4 (BB)
AR	100	100	0	0
AB	0	0	100	100
BR	100	0	100	0
BB	0	100	0	100

Compare AR and BB

	State 1 (RR)	State 2 (RB)	State 3 (BR)	State 4 (BB)
AR	100	100	0	0
AB	0	0	100	100
BR	100	0	100	0
BB	0	100	0	100

Preference between AR and BB should not change…

	State 1 (RR)	State 2 (RB)	State 3 (BR)	State 4 (BB)
AR	100	0	100	0
AB	0	0	100	100
BR	100	0	100	0
BB	0	0	100	100

But then we get BR and AB…

	State 1 (RR)	State 2 (RB)	State 3 (BR)	State 4 (BB)
AR	100	0	100	0
AB	0	0	100	100
BR	100	0	100	0
BB	0	0	100	100

The descriptive problem

People often do not behave as if there had subjective probabilities

- Investment in financial markets
- Fear of epidemics, financial crises, terrorist attacks…

The normative problem

- It is not obvious that it is rational to behave as if there were probabilities
- What's the probability that all Arbodytes are Cyclophines?
  - (I made up these words)
  - 50%-50% ? How about the converse? How about the two being disjoint? How about meta-Arbodytes?

The normative problem

- The main point: the Bayesian approach doesn't have the language to say "I don't have any idea"
- "I don't know" is countered by "How much do you not know? 0.75? 0.74?"

Why aren’t we convinced by Savage?

- Well, it depends to a large extent on the state space
- When it is theoretically constructed, the axioms are less compelling
- Even the two-urn example required construction
- In real-life examples we may never even observe a state

Non-Bayesian models

Pioneering work: Choquet Expected Utility by David Schmeidler

The idea: if "probabilities" reflect willingness to bet, maybe they shouldn't add up

David Schmeidler
(b. 1939)

Non-additive probabilities

aka "capacities"

Set functions (on events) that are monotone (with respect to set inclusion) but need not be additive

Thus, we can have

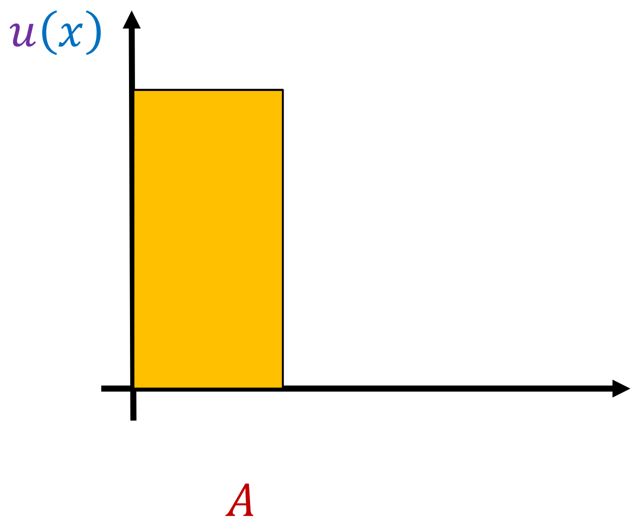
$$v(Red) = v(Black) = 0.3$$

while

$$v(Red \cup Black) = 1$$

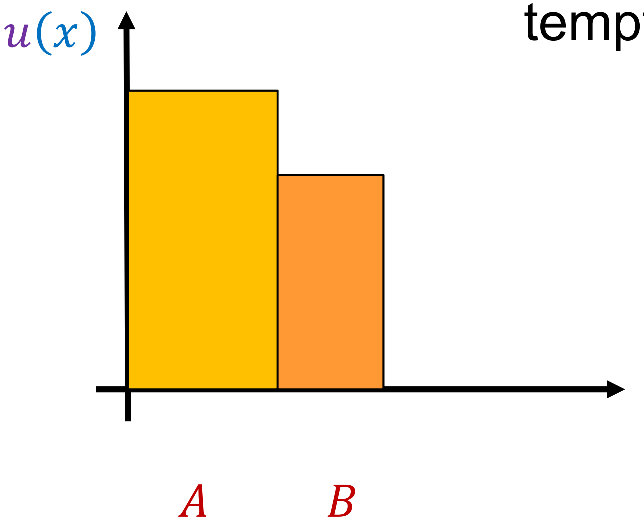
How do we integrate w.r.t. capacities?

With only two values, $u(x), 0$, and $u(x) > 0$



$v(A) * u(x_1)$

What about three values?

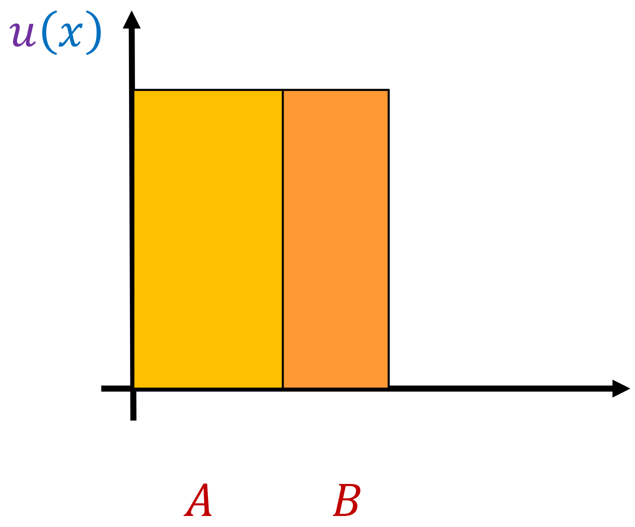


If we have x_1 on A and x_2 on B it's tempting to define

$$v(A) * u(x_1) + v(B) * u(x_2)$$

But there are problems…

First problem: definition



If $u(x_1) = u(x_2)$, do we use

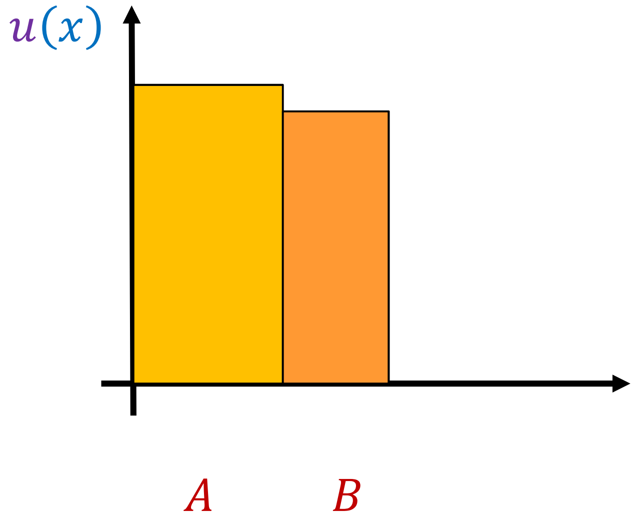
$$v(A) * u(x_1) + v(B) * u(x_2)$$

or

$$v(A \cup B) * u(x_1) \text{ ?}$$

… probably the latter (larger event), recalling that the state space may be infinite

Second problem: continuity



If $u(x_1) = u(x_2) + \varepsilon$

$$v(A) * u(x_1) + v(B) * u(x_2)$$

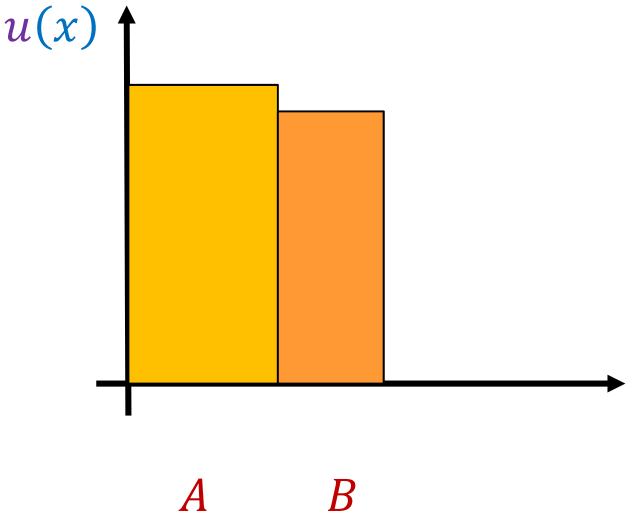
$$\rightarrow [v(A) + v(B)]\ u(x_2)$$

(as $\varepsilon \rightarrow 0$)

which is generally different than

$$v(A \cup B) * u(x_2)$$

Third problem: monotonicity



If $v(A) + v(B) < v(A \cup B)$

$$u(x_1) = u(x_2) + \varepsilon$$

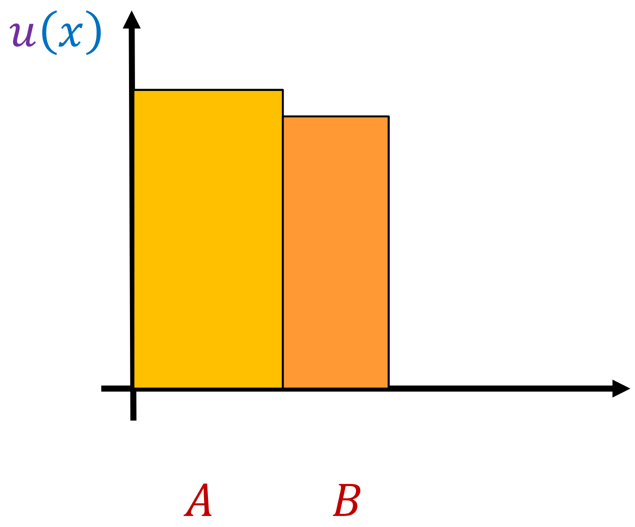
$$v(A) * u(x_1) + v(B) * u(x_2)$$

$$[\rightarrow [v(A) + v(B)]\ u(x_2)]$$

will be lower than (for a small $\varepsilon > 0$)

$$v(A \cup B) * u(x_2)$$

And, similarly,



If $v(A) + v(B) > v(A \cup B)$

$$u(x_2) = u(x_1) - \varepsilon$$

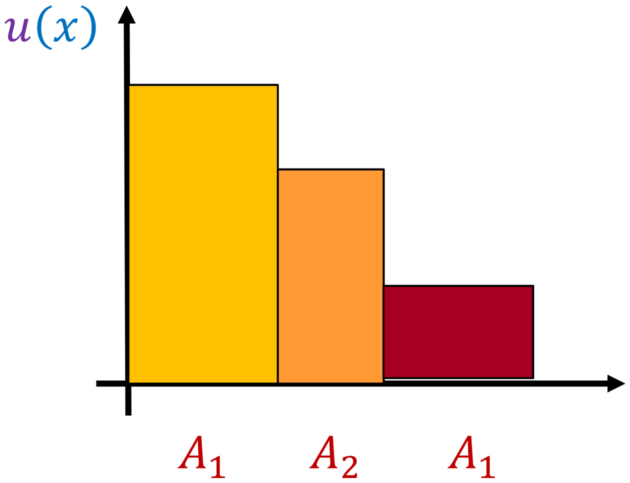
$$v(A) * u(x_1) + v(B) * u(x_2)$$

$$[\to [v(A) + v(B)]\ u(x_1)]$$

Will be higher than (for a small $\varepsilon > 0$)

$$v(A \cup B) * u(x_1)$$

Something isn't working



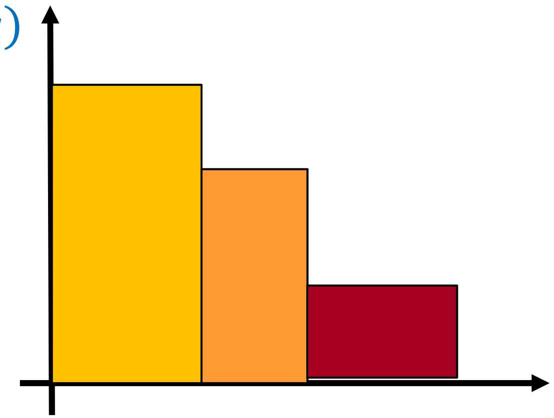
…with the Riemann-like definition:

$$v(A_1) * u(x_1)$$
$$+v(A_2) * u(x_2)$$
$$+v(A_3) * u(x_3)$$

But we could…

(With $u(x_1) \geq u(x_2) \geq u(x_3) \geq \cdots$)

Replace



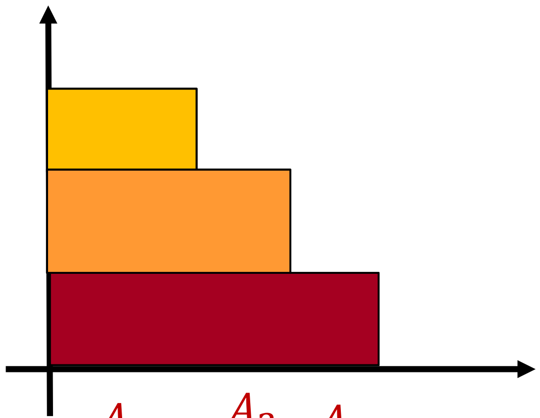
$v(A_1) * u(x_1)$

$+v(A_2) * u(x_2)$

$+v(A_3) * u(x_3)$

$+ \ldots$

by



$v(A_1) * [u(x_1) - u(x_2)]$

$+v(A_1 \cup A_2) * [u(x_2) - u(x_3)]$

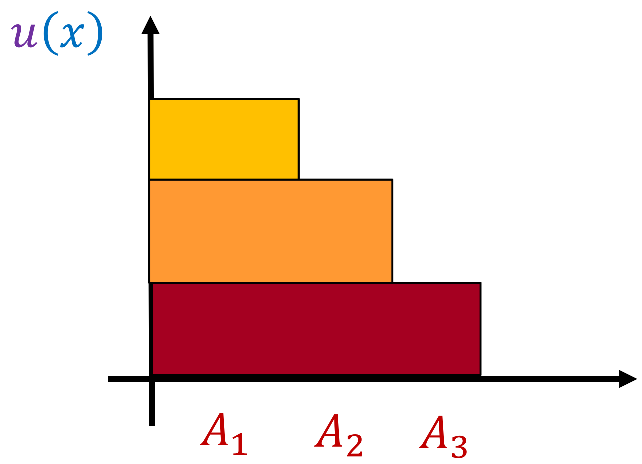
$+v(A_1 \cup A_2 \cup A_3) * [u(x_3) - u(x_4)]$

$+ \ldots$

Which is the Choquet integral

(With $u(x_1) \geq u(x_2) \geq u(x_3) \geq \cdots$)

$$v(A_1) * [u(x_1) - u(x_2)]$$
$$+v(A_1 \cup A_2) * [u(x_2) - u(x_3)]$$
$$+v(A_1 \cup A_2 \cup A_3) * [u(x_3) - u(x_4)]$$
$$+ \ldots$$
$$= v(A_1) * u(x_1)$$
$$+[v(A_1 \cup A_2) - v(A_1)] * u(x_2)$$
$$+[v(A_1 \cup A_2 \cup A_3) - v(A_1 \cup A_2)] * u(x_3)$$
$$+ \ldots$$



Looks familiar?

- Indeed, RDU is a special case of Choquet integration
- It is special in that there exist an underlying probability p and an increasing function f such that the capacity v is

$$v = f(p)$$

(that is, for every A , $v(A) = f(p(A))$)

- Note, however, that RDU can't deal with Ellsberg's paradoxes or other cases of ambiguity

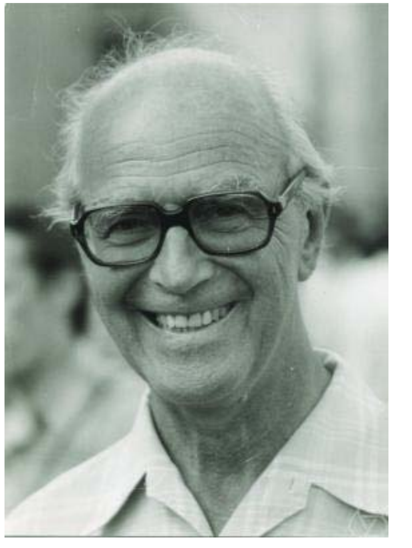
Choquet integral

Choquet introduced capacities and the integral in 1953

For $\varphi: S \to R$, $f \geq 0$,

$$\int \varphi dv = \int_0^\infty v(\varphi \geq t)dt$$

$$= \int_0^\infty v(\{s|\varphi(s) \geq t\})dt$$



Gustave Choquet (1915-2006)

The Choquet integral for step functions

$$\int \varphi dv = \int_0^\infty v(\varphi \geq t)dt =$$

$$v(A_1) * [\varphi_1 - \varphi_2]$$

$$+v(A_1 \cup A_2) * [\varphi_2 - \varphi_3]$$

$$+v(A_1 \cup A_2 \cup A_3) * [\varphi_3 - \varphi_4]$$

$$+ \ldots$$

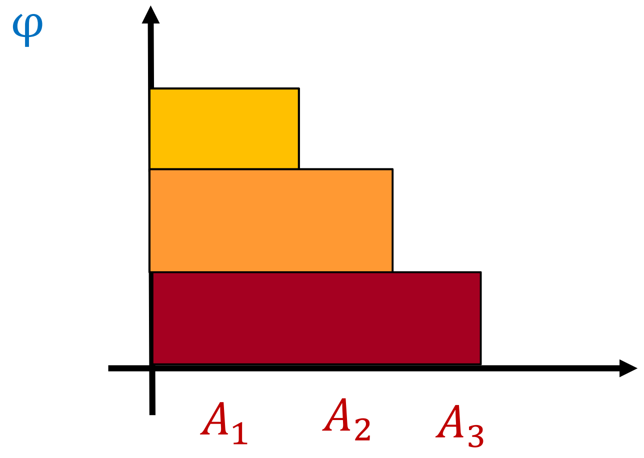
$$= v(A_1) * \varphi_1$$

$$+[v(A_1 \cup A_2) - v(A_1)] * \varphi_2$$

$$+[v(A_1 \cup A_2 \cup A_3) - v(A_1 \cup A_2)] * \varphi_3$$

$$+ \ldots$$

(With $\varphi_1 \geq \varphi_2 \geq \varphi_3 \geq \cdots$)



We had a problem with the Independence Axiom

There be two states $S = \{R, B\}$

Betting on Red gives the vector of winning probabilities $(1,0)$

Betting on Black gives the vector of winning probabilities $(0,1)$

They are equivalent

Independence means that they should be equivalent also to

$$\frac{1}{2}(1,0) + \frac{1}{2}(0,1) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

Why might we not satisfy Independence here?

The problem with the Independence Axiom

Why might we not satisfy Independence here?

… Because we start with the indifference

$$(1,0) \sim (0,1)$$

And "mix" them with $(1,0)$, so that we should get

$$(1,0) \sim \frac{1}{2}(1,0) + \frac{1}{2}(0,1) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

But when we mix $(0,1)$ with $(1,0)$ we hedge

This hedging obviously doesn't exist when we mix $(0,1)$ with itself

Comonotonicity

Two acts f, g are comonotonic if there do not exist two states s, t such that

$$f(s) \succ f(t)$$

but

$$g(s) \prec g(t)$$

Intuitively, they cannot hedge against each other

Schmeidler: let's restrict the independence axiom to mixing comonotonic acts

Anscombe-Aumann's Theorem

AA1. $\succcurlyeq$ on F is complete and transitive

AA2. $f \succ g \succ h \Rightarrow \exists\ \alpha, \beta \in (0,1)$ such that

$$\alpha f + (1-\alpha)h \succ g \succ \beta f + (1-\beta)h$$

AA3. $f \succ g \Rightarrow \forall\ \alpha > 0, \forall h,$

$$\alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$$

AA4. $[\forall\ s \quad f(s) \succcurlyeq g(s)] \Rightarrow f \succcurlyeq g$

AA5. There exist $f \succ g$

IFF

There exist a non-constant $u: X \to R$ (unique up to p.l.t.) and a (unique) measure p on S such that, for every f, g

$$f \succcurlyeq g \Leftrightarrow \int Eu\big(f(s)\big)dp \geq \int Eu\big(g(s)\big)dp$$

Schmeidler's Theorem

AA1. $\succcurlyeq$ on F is complete and transitive

AA2. $f \succ g \succ h \Rightarrow \exists\ \alpha, \beta \in (0,1)$ such that

$$\alpha f + (1-\alpha)h \succ g \succ \beta f + (1-\beta)h$$

Com. Ind. $f \succ g \Rightarrow \forall \alpha > 0, \forall h,$ comonotonic with both f, g

$$\alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$$

AA4.$[\forall\ s \quad f(s) \succcurlyeq g(s)] \Rightarrow f \succcurlyeq g$

AA5. There exist $f \succ g$

IFF

There exist a non-constant $u: X \to R$ (unique up to p.a.t.) and a (unique) capcaity v on S such that, for every f, g

$$f \succcurlyeq g \Leftrightarrow \int Eu\big(f(s)\big)dv \geq \int Eu\big(g(s)\big)dv$$

Reference

Subjective probability and expected utility without additivity

David Schmeidler

Econometrica, Vol. 57, No. 3 (May, 1989), pp. 571-587

Abstract

An act maps…

If a nondegenerate…

The extension of expected utility theory covers situations, as the Ellsberg Paradox, which are inconsistent with additive expected utility. The concept of *uncertainty aversion* and interpretation of comonotonic independence in the context of social welfare functions are included.

Other models of ambiguity

- There are other models than can capture "ambiguity aversion"
- For example: maxmin expected utility
- Rather than a single probability, there is a set of probabilities (as in Classical Statistics), and each act is evaluated by its worst-case expected utility

The Maxmin Theorem

AA1. $\succcurlyeq$ on F is complete and transitive

AA2. $f \succ g \succ h \Rightarrow \exists\ \alpha, \beta \in (0,1)$ such that

$$\alpha f + (1-\alpha)h \succ g \succ \beta f + (1-\beta)h$$

C-Ind. $f \succ g \Rightarrow \forall \alpha > 0, \forall h,$ constant

$$\alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$$

AA4.$[\forall\, s \quad f(s) \succcurlyeq g(s)] \Rightarrow f \succcurlyeq g$

AA5. There exist $f \succ g$

IFF

Uncertainty Aversion $f \sim g \Rightarrow 0.5f + 0.5g \succcurlyeq g$

There exist a non-constant $u: X \to R$ (unique up to p.a.t.) and a (unique) convex and closed subset C of $\Delta(S)$ such that, for every f, g

$$f \succcurlyeq g \Leftrightarrow min_{p \in C} \int Eu\big(f(s)\big)dp \geq min_{p \in C} \int Eu\big(g(s)\big)dp$$

Reference

Maxmin expected utility with non-unique prior

Itzhak Gilboa, David Schmeidler

Journal of Mathematical Economics, Vol. 18, No. 2 (1989), pp. 141-153

Abstract

Acts are functions from states of nature into finite-support distributions over a set of ‘deterministic outcomes’. We characterize preference relations over acts which have a numerical representation by the functional $J(f)=\min \{\int u\circ f dP \mid P\in C\}$ where *f* is an act, *u* is a von Neumann-Morgenstern utility over outcomes, and *C* is a closed and convex set of finitely additive probability measures on the states of nature. In addition to the usual assumptions on the preference relation as transitivity, completeness, continuity and monotonicity, we assume uncertainty aversion and certainty-independence. The last condition is a new one and is a weakening of the classical independence axiom: It requires that an act *f* is preferred to an act *g* if and only if the mixture of *f* and any constant act *h* is preferred to the same mixture of *g* and *h*. If non-degeneracy of the preference relation is also assumed, the convex set of priors *C* is uniquely determined. Finally, a concept of independence in case of a non-unique prior is introduced.

Maxmin EU

- Advantages:
  - Simple to explain
  - Related to statistics
  - Can be applied without an explicit state space
- Disadvantages:
  - Can't explain ambiguity liking
  - Seem very extreme

We're supposed to take issue with the axioms, but still…

The “smooth” model

Peter Klibanoff

Massimo Marinacci (b. 1965)

Sujoy Mukerji

Assume that there are second-order probabilities and that the expected utility values of each act (relative to different probabilities) are somehow integrated

The "smooth" model – main idea

Second-order probabilities μ on $\Delta(S)$

For an act f and a probability p we have expected utility

$$\int Eu\big(f(s)\big)dp$$

Instead of taking the worst-case

$$min_{p\in C} \int Eu\big(f(s)\big)dp$$

We can take some average relative to μ

The “smooth” model – formula

But if we just take

$$\int \left[\int Eu\big(f(s)\big)dp \;\right] d\mu$$

It will be like

$$\int Eu\big(f(s)\big)dp_0$$

for

$$p_0 = \int p d\mu$$

… and won’t be able to explain anything non-Bayesian

So KMM add a non-linear element

Maximize

$$\int \varphi \left[\int Eu(f(s))dp \right] d\mu$$

The model looks less extreme

And it can behave in a smooth way (as opposed to the “min”, which isn’t differentiable)

Reference

A smooth model of decision under ambiguity

Peter Klibanoff, Massimo Marinacci, Sujoy Mukerji

Econometrica, Vol. 73, No. 6 (Nov, 2005), pp. 1849-1892

Abstract

We propose and characterize a model of preferences over acts such that the decision maker prefers act f to act g if and only if $\mathbb{E}_{\mu}\varphi(\mathbb{E}_{\pi}u\circ f)$ $\mathbb{E}_{\mu}\varphi(\mathbb{E}_{\pi}u\circ g)$, where $\mathbb{E}$ is the expectation operator, u is a von Neumann–Morgenstern utility function, φ is an increasing transformation, and μ is a subjective probability over the set Π of probability measures π that the decision maker thinks are relevant given his subjective information. A key feature of our model is that it achieves a separation between ambiguity, identified as a characteristic of the decision maker's subjective beliefs, and ambiguity attitude, a characteristic of the decision maker's tastes. We show that attitudes toward pure risk are characterized by the shape of u, as usual, while attitudes toward ambiguity are characterized by the shape of φ. Ambiguity itself is defined behaviorally and is shown to be characterized by properties of the subjective set of measures Π. One advantage of this model is that the well-developed machinery for dealing with risk attitudes can be applied as well to ambiguity attitudes. The model is also distinct from many in the literature on ambiguity in that it allows smooth, rather than kinked, indifference curves. This leads to different behavior and improved tractability, while still sharing the main features (e.g., Ellsberg's paradox). The maxmin expected utility model (e.g., Gilboa and Schmeidler (1989)) with a given set of measures may be seen as a limiting case of our model with infinite ambiguity aversion. Two illustrative portfolio choice examples are offered.

Survey

Probability and uncertainty in economic modeling

Itzhak Gilboa, Andrew W. Postlewaite, David Schmeidler

Journal of Economic Perspectives, Vol. 22, No. 3 (Summer, 2008), pp. 173-188

Abstract

Economic modeling assumes, for the most part, that agents are Bayesian, that is, that they entertain probabilistic beliefs, objective or subjective, regarding any event in question. We argue that the formation of such beliefs calls for a deeper examination and for explicit modeling. Models of belief formation may enhance our understanding of the probabilistic beliefs when these exist, and may also help us characterize situations in which entertaining such beliefs is neither realistic nor necessarily rational.

Survey

Ambiguity and the Bayesian paradigm

Itzhak Gilboa, Massimo Marinacci

Abstract

This is a survey of some of the recent decision-theoretic literature involving beliefs that cannot be quantified by a Bayesian prior. We discuss historical, philosophical, and axiomatic foundations of the Bayesian model, as well as of several alternative models recently proposed. The definition and comparison of ambiguity aversion and the updating of non-Bayesian beliefs are briefly discussed. Finally, several applications are mentioned to illustrate the way that ambiguity (or “Knightian uncertainty”) can change the way we think about economic problems.

Other explanations of Ellsberg's Paradoxes

People fail to reduce

compound lotteries

(Reduction of compound lotteries is

implicitly assumed in the formulation

of Anscombe-Aumann we used here)

Uzi Segal

Other explanations of Ellsberg's Paradoxes

People mistakenly think that

the problem is repeated

And then risk aversion suffices to

explain their behavior

Yoram Halevy

Reference

The Ellsberg paradox and risk aversion: an anticipated utility approach

Uzi Segal

International Economic Review, Vol. 28, No. 1 (1987), pp. 175-202

Abstract

The paper describes a decision process under which it is rational to prefer a lottery with known probabilities to a similar ambiguous lottery where the decision maker does not know the exact values of the probabilities (the “Ellsberg paradox”). This is done by modeling ambiguous lotteries as two-stage lotteries, by assuming the independence axiom without the reduction of compound lotteries axiom, and by using the anticipated utility functional. This paper also gives conditions under which less ambiguity is preferred and presents some comparative statics analysis as well as some inter-personal comparisons. Finally, it proves that within the anticipated utility framework, risk and ambiguity are almost identical.

See

Ellsberg revisited: An experimental study

Yoram Halevy

Econometrica, Vol. 75, No. 2 (2007), pp. 503-536

Abstract

An extension to Ellsberg's experiment demonstrates that attitudes to ambiguity and compound objective lotteries are tightly associated. The sample is decomposed into three main groups: subjective expected utility subjects, who reduce compound objective lotteries and are ambiguity neutral, and two groups that exhibit different forms of association between preferences over compound lotteries and ambiguity, corresponding to alternative theoretical models that account for ambiguity averse or seeking behavior.

Reference

A Bayesian approach to uncertainty aversion

Yoram Halevy, Vincent Feltkamp

The Review of Economic Studies, Vol. 72, No. 2 (2005), pp. 449-466

Abstract

The Ellsberg paradox demonstrates that people's beliefs over uncertain events might not be representable by subjective probability. We show that if a risk averse decision maker, who has a well defined Bayesian prior, perceives an Ellsberg type decision problem as possibly composed of a bundle of several positively correlated problems, she will be uncertainty averse. We generalize this argument and derive sufficient conditions for uncertainty aversion.

States of the World

Monty Hall's 3-Door Problem

Consider the following version of the TV game "Let's Make a Deal": there are three doors, marked A, B, and C, and behind one of them there is a prize (a car). Behind the two other doors there is no prize (a goat). Based on past plays of the game, you can safely assume that the car is behind doors A, B, and C with equal probabilities.

You are asked to name a door. Before you open it, the moderator (Monty Hall), who knows where the car is, opens a door. He has to open a door that (i) differs from the one you named; and (ii) does not have the car behind it. (Since there are three doors, he can always do that.) Now you are given a choice: you can either open the first door you named ("stick") or open the other door still closed ("switch"). You get the prize behind the door you decide to open, and you goal is to maximize the probability of getting the car. What should you do?

It’s better to switch

The probability of getting the car

Car is…	A	B	C
Stick	1	0	0
Switch	0	1	1

To be convinced

Assume that there were 1000 doors (and he has to open 998)

We realize that “switching” is not always to the same door

Monty Hall “helps” us by ruling out one of the bad choices we could have made.

But…

What's wrong with the simple calculation:

State (car is behind…)	A	B	C
Prior	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Posterior (after opening B)	$\frac{1}{2}$	0	$\frac{1}{2}$

The problem

The state space above $\{A, B, C\}$ is not rich enough

Assumptions that are hidden in the definition of the states will never be challenged by Bayesian updating

In particular, the way we get information, and the fact that we know something, may be informative in itself.

A correct Bayesian analysis

- Suppose we named door A

- Two sources of uncertainty
  - Where the car really is
  - Which door Monty Hall opens

- Thus, there are 9 states of the world, not 3:

The states of the world

MH opens/ car is behind…	A	B	C	Marginal
A				
B				
C				
Marginal				

Filling up probabilities

MH opens/ car is behind…	A	B	C	Marginal
A	0			
B	0			
C	0			
Marginal				

Filling up probabilities

MH opens/ car is behind…	A	B	C	Marginal
A	0			
B	0	0		
C	0		0	
Marginal				

Filling up probabilities

MH opens/ car is behind…	A	B	C	Marginal
A	0			$\frac{1}{3}$
B	0	0		$\frac{1}{3}$
C	0		0	$\frac{1}{3}$
Marginal				1

Filling up probabilities

MH opens/ car is behind…	A	B	C	Marginal
A	0			$\frac{1}{3}$
B	0	0	$\frac{1}{3}$	$\frac{1}{3}$
C	0	$\frac{1}{3}$	0	$\frac{1}{3}$
Marginal				1

Filling up probabilities

MH opens/ car is behind…	A	B	C	Marginal
A	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
B	0	0	$\frac{1}{3}$	$\frac{1}{3}$
C	0	$\frac{1}{3}$	0	$\frac{1}{3}$
Marginal	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Conditioning on "not B"

MH opens/ car is behind…	A	B	C	Marginal
A	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
B	0	0	$\frac{1}{3}$	$\frac{1}{3}$
C	0	$\frac{1}{3}$	0	$\frac{1}{3}$
Marginal	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Conditioning on “not B”

MH opens/ car is behind…	A	B	C	Marginal
A	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1/3}{2/3} = \frac{1}{2}$
B	0	0	$\frac{1}{3}$	$\frac{1}{3}$
C	0	$\frac{1}{3}$	0	$\frac{1/3}{2/3} = \frac{1}{2}$
Marginal	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Conditioning on "MH opened B"

MH opens/ car is behind…	A	B	C	Marginal
A	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
B	0	0	$\frac{1}{3}$	$\frac{1}{3}$
C	0	$\frac{1}{3}$	0	$\frac{1}{3}$
Marginal	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Conditioning on "MH opened B"

MH opens/ car is behind…	A	B	C	Marginal
A	0	$\frac{1/6}{1/2} = \frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$
B	0	0	$\frac{1}{3}$	$\frac{1}{3}$
C	0	$\frac{1/3}{1/2} = \frac{2}{3}$	0	$\frac{1}{3}$
Marginal	0	$\frac{1}{2}$	$\frac{1}{2}$	1

And what if…

MH opens/ car is behind…	A	B	C	Marginal
A	0	α	$\frac{1}{3} - \alpha$	$\frac{1}{3}$
B	0	0	$\frac{1}{3}$	$\frac{1}{3}$
C	0	$\frac{1}{3}$	0	$\frac{1}{3}$
Marginal	0	$\frac{1}{3} + \alpha$	$\frac{2}{3} - \alpha$	1

It is important, however…

MH opens/ car is behind…	A	B	C	Marginal
A	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
B	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{3}$
C	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{3}$
Marginal	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1

A problem: cutting prices?

- If the competitor doesn’t, we can get he entire market
- If it does, we surely should, not to be left out
- It seems that cutting prices is a dominant strategy
- A sort of Prisoner’s Dilemma

If the decisions are independent…

It is indeed a PD

State of world/ act	Competitor cuts prices	Competitor doesn't cut prices
Cut Prices	L, L	L, H
Not Cut Prices	H, L	H, H

And yet…

- Maybe the decisions are not independent?

- After all, the decision "not to cut prices" doesn't happen at a given t

An alternative analysis

- The states of the world should reflect all possible causal relationships
- In order to assume nothing a-priori, we define states as functions from acts to outcomes:

The states of the world

What will result if we…	Cut	Not Cut
State 1	Cut	Cut
State 2	Cut	Not Cut
State 3	Not Cut	Cut
State 4	Not Cut	Not Cut

The new decision matrix

	State 1	State 2	State 3	State 4
	(C, C)	(C, NC)	(NC, C)	(NC, NC)
Cut	L, L	L, L	L, H	L, H
Not Cut	H, L	H, H	H, L	H, H

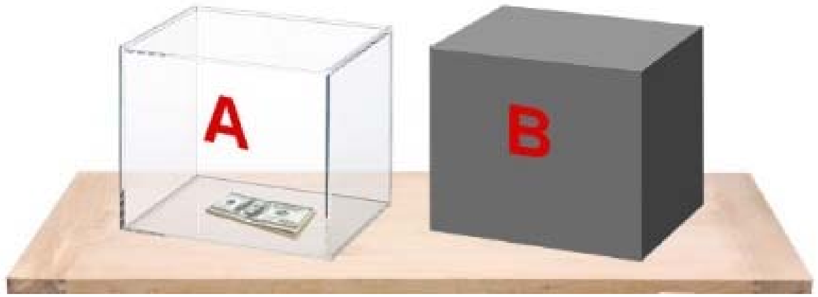
Newcomb's Paradox

The above was a variant of Newcomb's paradox:

There are two boxes

A – transparent with $\$1{,}000$

B – opaque with $\$1M$ or $\$0$



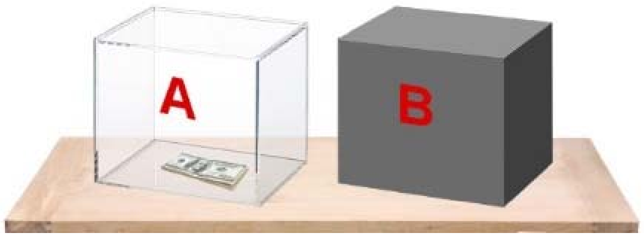
You're asked to choose between Only B or Both

Why not Both?

Well, an omniscient observer put the $\$1M$ in B if and only if you're modest and take Only B

Hmmm…

A simpler story: this is what the data show



The point

Choosing Both appears to be dominant if one assumes that there is causal independence between one's choice and the $\$1M$ being in B

But the data suggest that this causal independence doesn't hold. We need to re-define the states.

Gibbard and Harper (1978)

Reference

Counterfactuals and two kinds of expected utility

Allan Gibbard, William Harper

In A. Hooker, J. J. Leach & E. F. McClennen (eds.), *Foundations and Applications of Decision Theory*. D. Reidel. pp. 125-162 (1978)

Abstract

We begin with a rough theory of rational decision-making. In the first place, rational decision-making involves conditional propositions: when a person weighs a major decision, it is rational for him to ask, for each act he considers, what would happen if he performed that act. It is rational, then, for him to consider propositions of the form 'If I were to do a, then c would happen'. Such a proposition we" shall call a counterfactual, and we shall form counter-factuals with a connective '□→' on this pattern: 'If I were to do a, then c would happen' is to be written 'I do a □→ c happens'.

Important lessons

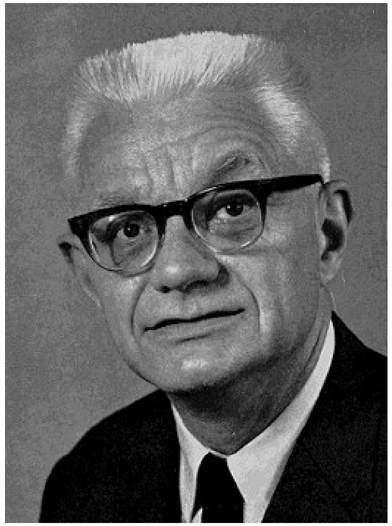
- The states of the world should specify all the information that might be relevant, including how information was given
- The states of the world should allow for all causal relationships

- Other examples: political discussions

PUZZLES OF INDUCTION

Hempel’s paradox of confirmation

Suppose that you want to test the hypothesis that All ravens are black



Carl Gustav Hempel (1905-1997)

You sample ravens, and if they are indeed black, you feel that the theory got some confirmation

But, says Hempel,

"All ravens are black"

Is the same as

"All that isn't black isn't a raven"

That is

$$(R \rightarrow B) \Leftrightarrow (\neg B \rightarrow \neg R)$$

This is the "contrapositive"

Contrapositives

We know that people get confused between

$$A \to B \text{ and } B \to A$$

The above are not the same!

But if we switch to negation when we change the direction of implication, that’s not a mistake:

$$(A \to B) \Leftrightarrow (\neg B \to \neg A)$$

Indeed,

$$(A \to B) \equiv (\neg A \vee B)$$

and both

$$A \to B \text{ and } \neg B \to \neg A$$

Simply mean that we can't have A and not B

	B	$\neg B$
A		
$\neg A$		

Back to Hempel

So

"All ravens are black"

is equivalent to

"All that isn't black isn't a raven"

That is

$$(R \rightarrow B) \Leftrightarrow (\neg B \rightarrow \neg R)$$

Then how about this slide?

It isn't black

Upon careful inspection, it isn't a raven either

Does it confirm the hypothesis that all ravens are black?

Quantitative issues

There are more non-black objects than there are ravens

Yes, but:

- When sampling with replacement, we anyway think of the population as infinite
- And we might want to take cost into account, too
- And let’s be serious…

Contrapositives are real

I could imagine testing non-black objects and looking for ravens

Suppose I can’t tell whether a bird is or isn’t a raven

I could sample $1{,}000{,}000$ non-black birds and test them for ravenhood

So?

A careful Bayesian analysis shows where the trick lies

A – objects

Each $i \in A$ can be a raven or not, black or not:

	$\neg Black$	$Black$
$\neg Raven$	$(0,0)$	$(0,1)$
$Raven$	$(1,0)$	$(1,1)$

States of the world

… each resolves all uncertainty:

$$\omega : A \to \{(0,0), (0,1), (1,0), (1,1)\}$$

or

$$\omega : A \to \{0,1\}^2$$

The state space consists of all of these:

$$\Omega = \{\omega \mid \omega : A \to \{0,1\}^2\}$$

The theory of interest

$$Q_i = \{\omega \mid \omega : A \to \{0,1\}^2, \omega(i) \neq (1,0)\}$$

That is, object i isn't a counterexample

$$Q \equiv \cap_{i \in A} Q_i$$

Namely, there are no counterexamples. All ravens are indeed black

Now assume that we tested i and Q_i has been observed. What will happen to our belief in the theory?

Bayesian updating

$$P(Q|Q_i) = \frac{P(Q \cap Q_i)}{P(Q_i)} = \frac{P(Q)}{P(Q_i)}$$

(Because $Q \equiv \bigcap_{i \in A} Q_i$ we have $Q \cap Q_i = Q$)

Now, will

$$\frac{P(Q)}{P(Q_i)} > P(Q)$$

Well, if and only if $P(Q_i) < 1$

A resolution to Hempel's paradox

$$\frac{P(Q)}{P(Q_i)} > P(Q)$$

if and only if $P(Q_i) < 1$

We get no confirmation to "all ravens are black" from the non-black slide because we had zero probability that it would turn out a counter-example

Because we knew it wasn't a raven

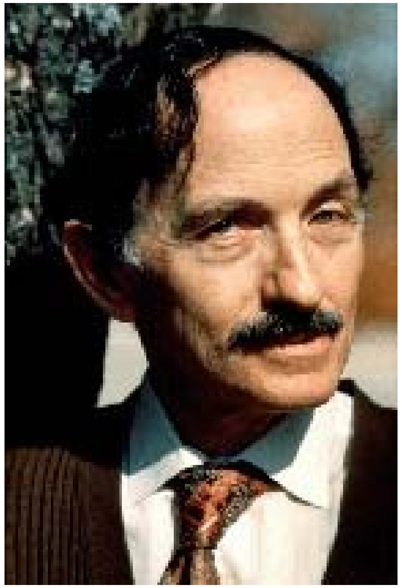
(Had we not been sure, we would have gotten some confirmation)

If we're so smart…

Good asked (something along the lines of):

Assume that the world has two objects, which are either both white swans, or both ravens, in which case one is black and one is white.

What happens when you sample an object and get a black raven?



Irving John Good
(1916-2009)

Bayesian beliefs

Let's say the two are equally likely:

$\{WS, WS\}$ with prob. 50%

$\{BR, WR\}$ with prob. 50%

A priori my belief in "All ravens are black" is 50%

(as it is true in the first world but not in the second)

(The example doesn't rely on zeros, which are only my simplification)

After having sampled a black raven, it goes down to 0%

(as I know I'm in the second world)

What happened to

$$P(Q|Q_i) = \frac{P(Q \cap Q_i)}{P(Q_i)} = \frac{P(Q)}{P(Q_i)}$$

$$\frac{P(Q)}{P(Q_i)} \geq P(Q)$$

How can the posterior belief be lower than the prior after all?

The trick

Good sneaks in some extra information

Specifically, it is not only that we learned that object i isn't a counterexample

(for which it need not be a raven at all!)

– we also learned that there are ravens.

The states of the world

$Obj2$ / $Obj1$	WS	BS	WR	BR
WS	0.5	0	0	0
BS	0	0	0	0
WR	0	0	0	0.25
BR	0	0	0.25	0

All ravens are black

$Obj2$ / $Obj1$	WS	BS	WR	BR
WS	0.5	0	0	0
BS	0	0	0	0
WR	0	0	0	0.25
BR	0	0	0.25	0

The prior of “All ravens are black” is 50%

Given rows 1,2,4

$Obj2$ / $Obj1$	WS	BS	WR	BR
WS	0.5	0	0	0
BS	0	0	0	0
WR	0	0	0	0.25
BR	0	0	0.25	0

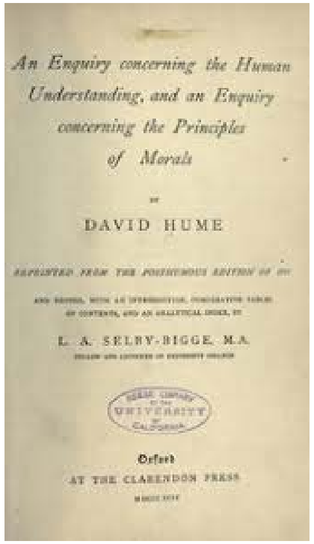
The prior 50% goes up to $\frac{2}{3}$

Given the fourth row

$Obj2$ / $Obj1$	WS	BS	WR	BR
WS	0.5	0	0	0
BS	0	0	0	0
WR	0	0	0	0.25
BR	0	0	0.25	0

The prior 50% goes down to 0

The Problem of Induction


An Enquiry concerning the Human Understanding, and an Enquiry concerning the Principles of Morals

DAVID HUME

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Oxford

AT THE CLARENDON PRESS

David Hume (1711-1776)

“That the sun will not rise tomorrow is no less intelligible a proposition, and implies no more contradiction, than the affirmation, that it will rise.” (Enquiry, 1748)

Justifying induction

Was a problem for Hume and remained so

W.V.O. Quine : “The human condition is the Humean condition”

Willard Van Orman Quine (1908-2000)

A puzzle

Continue the sequence

$$1\,,2\,,4\,,8\,,\_\_$$

A puzzle

Did you try

$$1\,,2\,,4\,,8\,,16\ ?$$

Why not

$$1\,,2\,,4\,,8\,,7\ \ ?$$

Both follow patterns

$$1, 2, 4, 8, 16$$

is predicted by

$$f(n) = 2^{n-1}$$

and

$$1, 2, 4, 8, 7$$

by

$$g(n) = -\frac{1}{3}n^4 + \frac{7}{2}n^3 - \frac{73}{6}n^2 + 18n - 8$$

Do you think that

$$f(n) = 2^{n-1}$$

is simpler than

$$g(n) = -\frac{1}{3}n^4 + \frac{7}{2}n^3 - \frac{73}{6}n^2 + 18n - 8$$?

… So it's a matter of bounded rationality?

And that's what we use on IQ tests?

The point

Without any subjective input, such as preference for simplicity, we are doomed to overfit

And not be able to predict simple patterns even if they are right before our eyes

If cognitive limitations did not exist, someone would have needed to invent them

See

Subjectivity in inductive inference

Itzhak Gilboa, Larry Samuelson

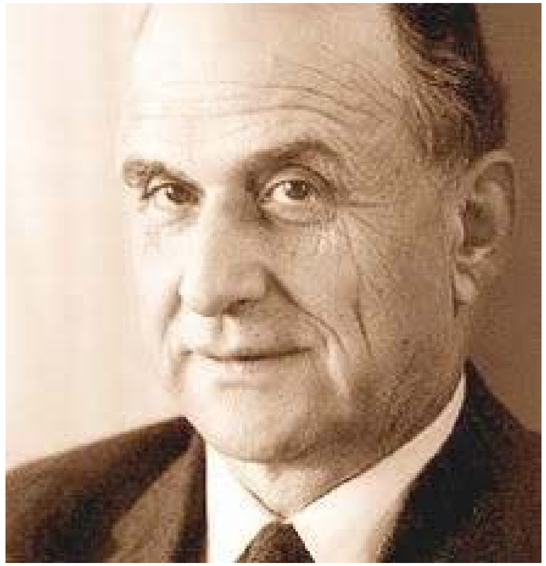
Theoretical Economics, Vol. 7 (2012), pp. 183-215

Abstract

This paper examines circumstances under which subjectivity enhances the effectiveness of inductive reasoning. We consider agents facing a data-generating process who are characterized by inference rules that may be purely objective (or data-based) ormay incorporate subjective considerations. Agents who invoke no subjective considerations are doomed to ineffective learning. The analysis places no computational or memory limitations on the agents: the role for subjectivity emerges in the presence of unlimited reasoning powers.

But it doesn't end there

Goodman posed the following question: if we want to tell whether emeralds are green or (say) blue, we test a sample and expect future observations to be in line with past ones.



Nelson Goodman (1906-1998)

So far so good.

Goodman's "New Riddle of Induction"

Next suppose that there are two types of emeralds:

$$grue = \begin{cases} green & t \leq T_0 \\ blue & t > T_0 \end{cases}$$

$$bleen = \begin{cases} blue & t \leq T_0 \\ green & t > T_0 \end{cases}$$

(Put T_0 in the future)

If we now test a sample, we'll conclude they are all grue,

and will all look blue to our eyes after T_0

Why would they change?

Why on earth would the green turn to blue at T_0 ?

Oh, they won't, says Goodman. Grue they are and grue they will remain.

On the contrary: Goodman asks, if they were grue all this time, why on earth would they change to bleen?

That is,

Goodmand suggests

$$green = \begin{cases} grue & t \leq T_0 \\ bleen & t > T_0 \end{cases}$$

$$blue = \begin{cases} bleen & t \leq T_0 \\ grue & t > T_0 \end{cases}$$

and argues for a complete symmetry between the green/blue and the grue/bleen categorizations

Indeed

The symmetry works if we can only refer to general terms and not to specific observations

That is, when an observation is an instantiation of a general concept

EMOTIONS and

STRATEGIC REASONING

Emotions and Rationality

The Ultimatum Game

There is a sum of $\$100$ to share between Players I and II

Player I offers a way to divide the sum (say, integer values)

Player II can say Yes or No

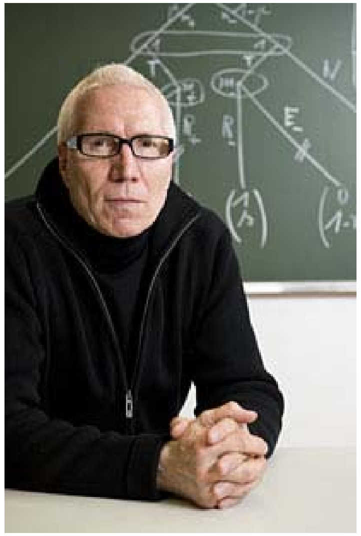
Yes – they get the amounts offered

No – they both get nothing

What will happen?

What does the theory say?

The Ultimatum Game



Werner Güth (b. 1944)

Bernd Schwarze (b. 1944)

Güth, Schmittberger, Schwarze (1982)

Reference

An experimental analysis of ultimatum bargaining

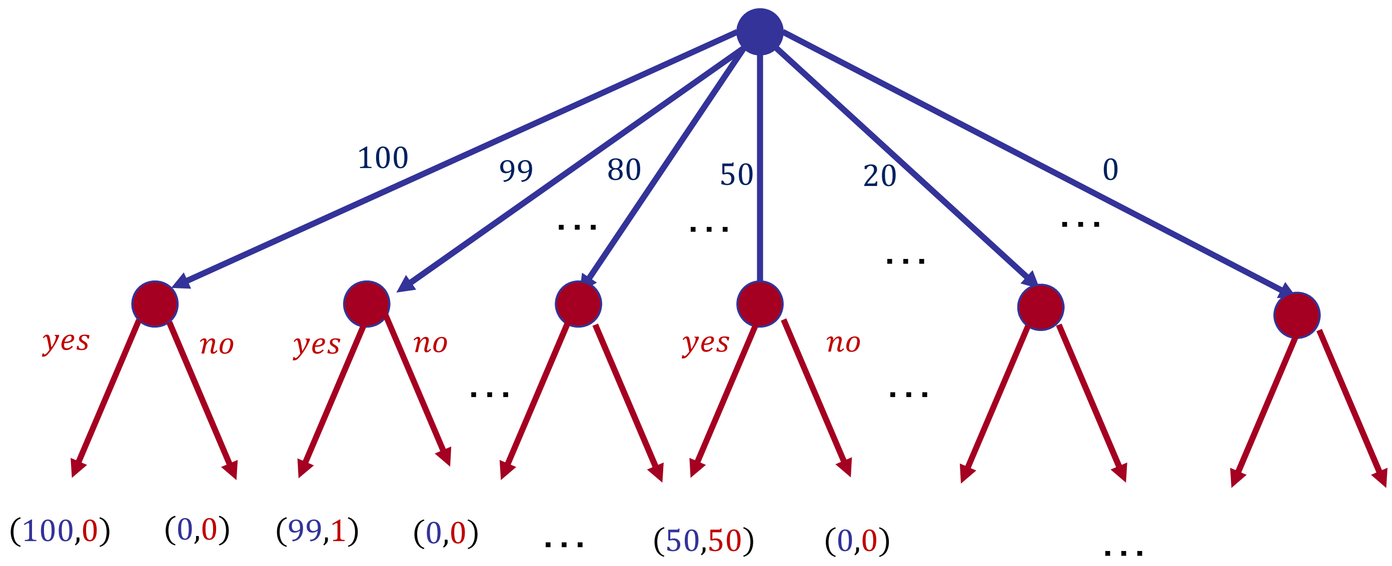
Werner Güth, Rolf Schmittberger, Bernd Schwarze

Journal of Economic Behavior and Organization, Vol. 3, No. 4 (Dec., 1982), pp. 367-388

Abstract

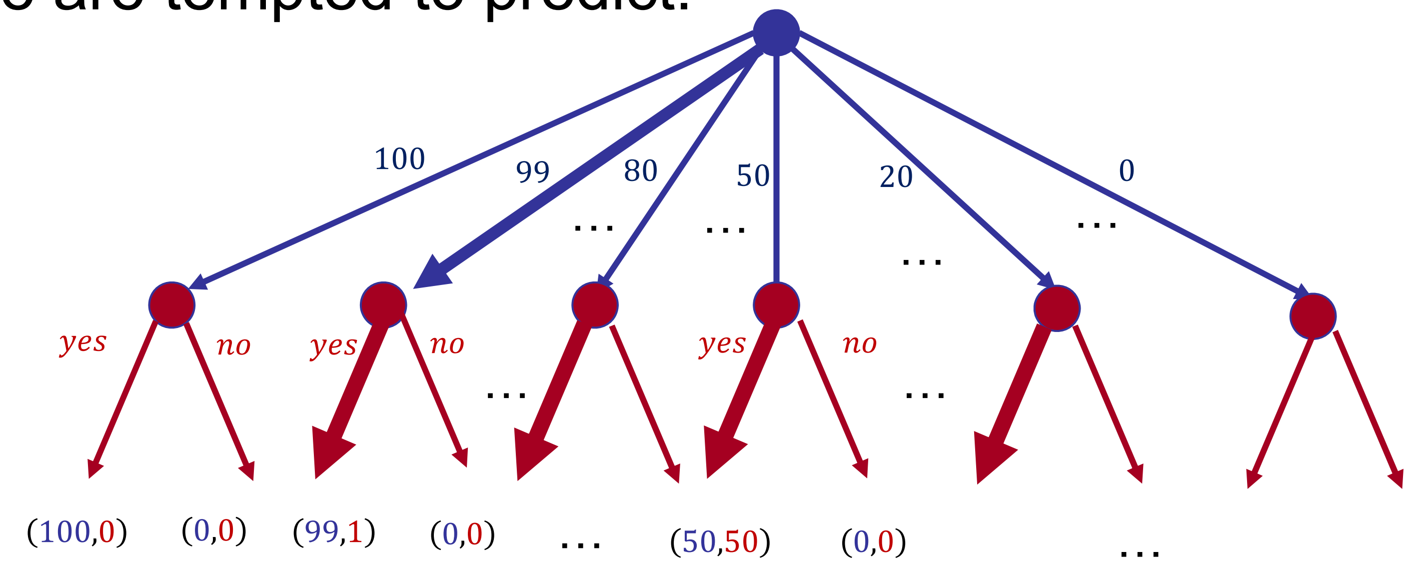
There are many experimental studies of bargaining behavior, but suprisingly enough nearly no attempt has been made to investigate the so-called ultimatum bargaining behavior experimentally. The special property of ultimatum bargaining games is that on every stage of the bargaining process only one player has to decide and that before the last stage the set of outcomes is already restricted to only two results. To make the ultimatum aspect obvious we concentrated on situations with two players and two stages. In the 'easy games' a given amount *c* has to be distributed among the two players, whereas in the 'complicated games' the players have to allocate a bundle of black and white chips with different values for both players. We performed two main experiments for easy games as well as for complicated games. By a special experiment it was investigated how the demands of subjects as player 1 are related to their acceptance decisions as player 2.

What does the theory say?



Well,

We are tempted to predict:



The “Backward Induction” solution

Backward Induction

In a finite game of perfect information we can go down to the leaves and work our way backwards to find the players' choices

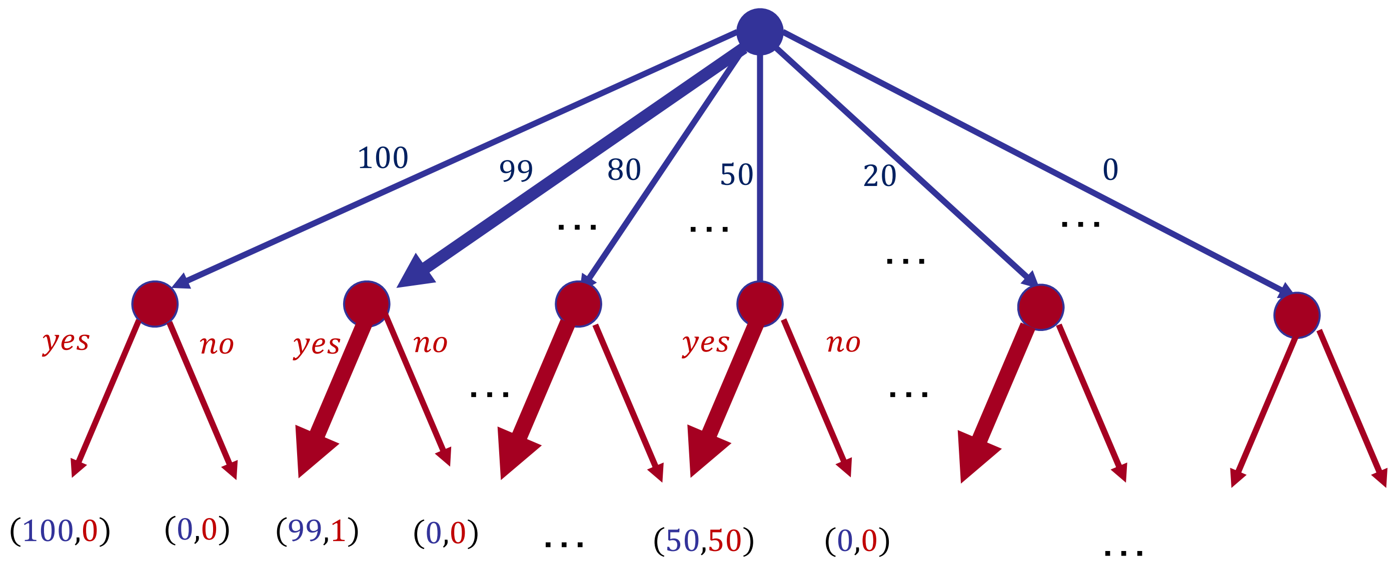
Ernest Zermelo (1871-1953)

Backward Induction assumptions

- Rationality
- Common knowledge (or common belief) in rationality
- To be precise, as many levels of belief as there are steps in the game

So in this case

The backward induction seems to be



– But this assumes that the monetary sums are the "utilities"

Important

- In a game as simple as the Ultimatum Game, it is impossible to test basic decision/game theoretic assumptions (such as transitivity)

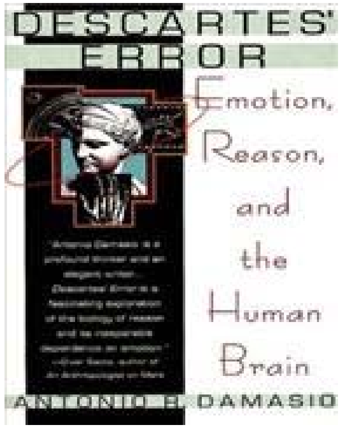
- We can only test them coupled with the assumption that only material payoffs matter

Emotional payoffs

- Player II might be angry/insulted at a low offer
- Player II as well as Player I might care for fairness
- Player I might be altruistic
- etc.

- A way to tell some explanations apart: the Dictator Game

Is it rational to respond to emotions?



Antonio Damasio (b. 1944)

In "Descartes' Error" (1994) argued that it is wrong to think of emotions and rationality as divorced; rather, rationality relies on emotions

How could we think otherwise?

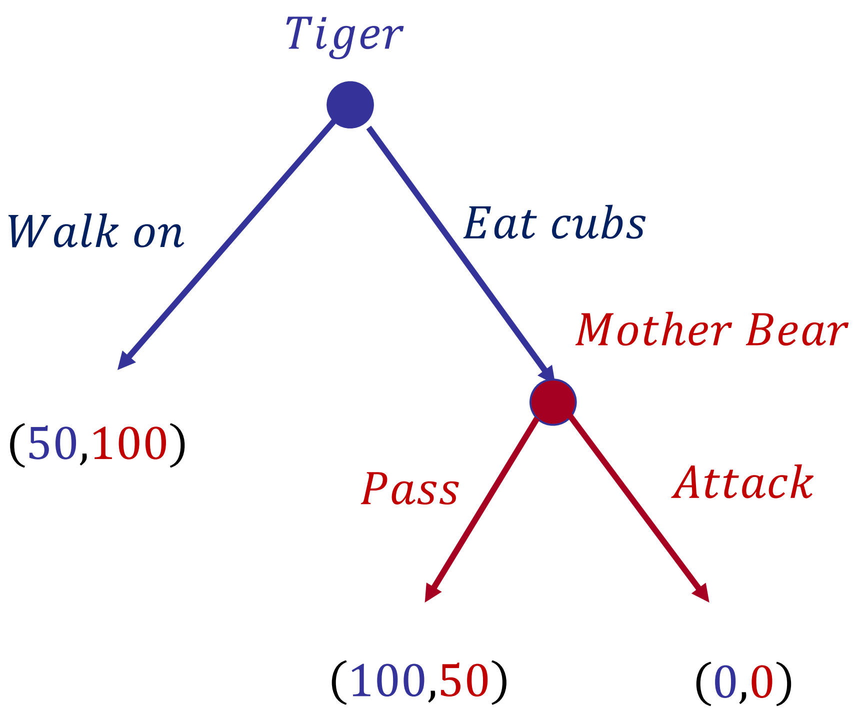
- Surely, there are cases where emotional reactions are excessive, "take over"
  - I may shoot someone and regret it for the rest of my life
- But pure reason cannot tell me what to do if I don't have an affective/hedonic reaction to it
- How would I "know" I should maximize u ?

Evolutionary Psychology

- Love of our children
- Caring for others (partners, siblings…)
- What about negative emotions?
- Who needs anger? Vindictiveness?

A toy model of anger

Material* payoffs

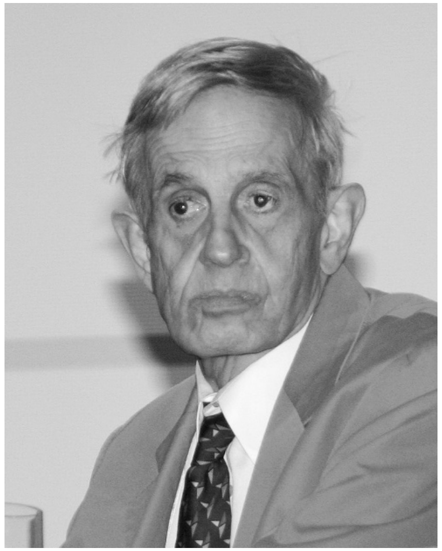


* Assuming maternal love but no vengeance

Nash equilibrium

A selection of a strategy for each player such that each is a best response to the others

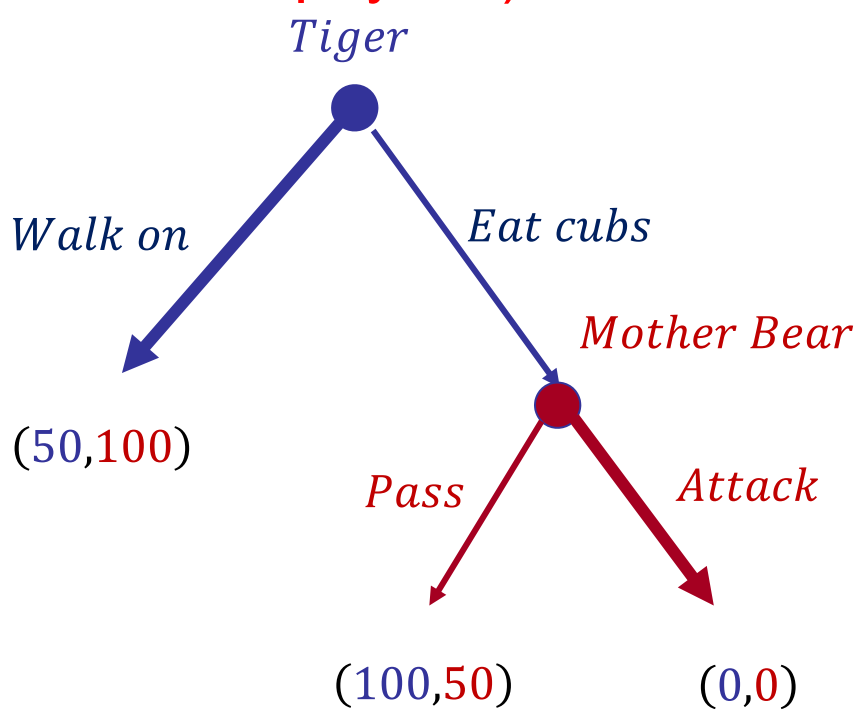
A deep theorem about existence in mixed strategies



John F. Nash (1928-2015)

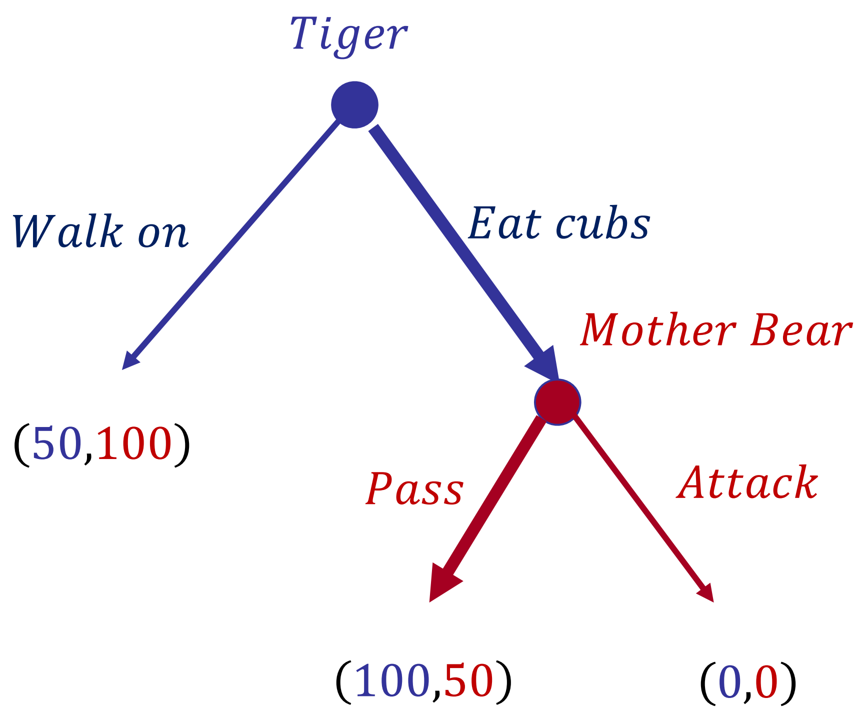
Nash equilibrium I

(in this game with material payoffs)



Nash equilibrium II

(in this game with material payoffs)



The game in the strategic form

		Mother Bear	
		Pass	*Attack*
Tiger	*Eat cubs*	100 , 50	0 , 0
	Walk on	50 , 100	50 , 100

Both shaded entries are Nash equilibria

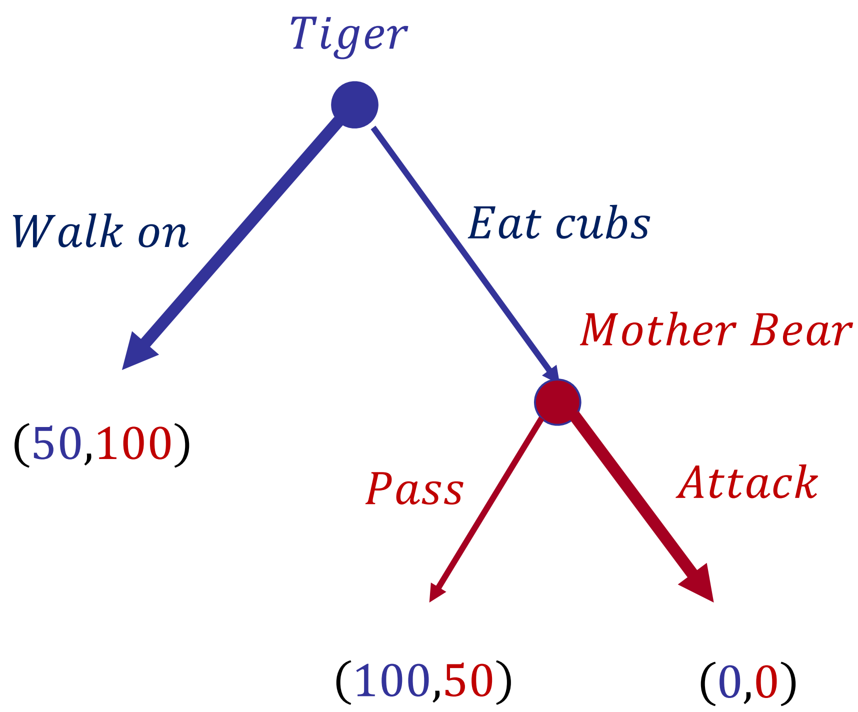
Subgame Perfect equilibrium

A Nash equilibrium that is still an equilibrium when we look at its restriction to any subgame

Reinhard Selten (1930-2016)

The idea: cross out threats that are not credible

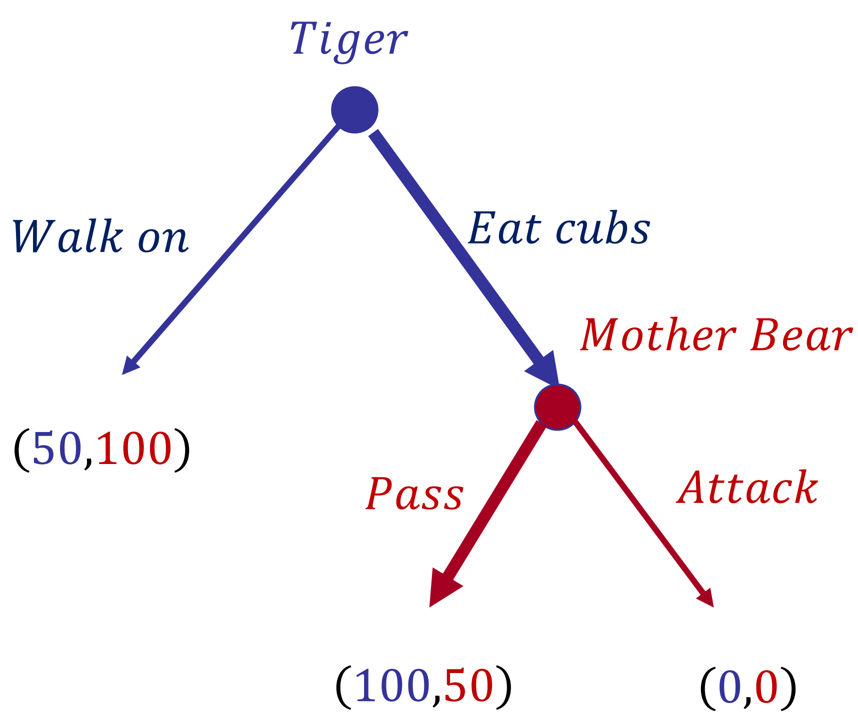
Equilibrium I isn't subgame perfect



Mother Bear's choice isn't optimal in the subgame

It is a non-credible threat

Equilibrium II is subgame perfect



As would be the Backward Induction solution in general

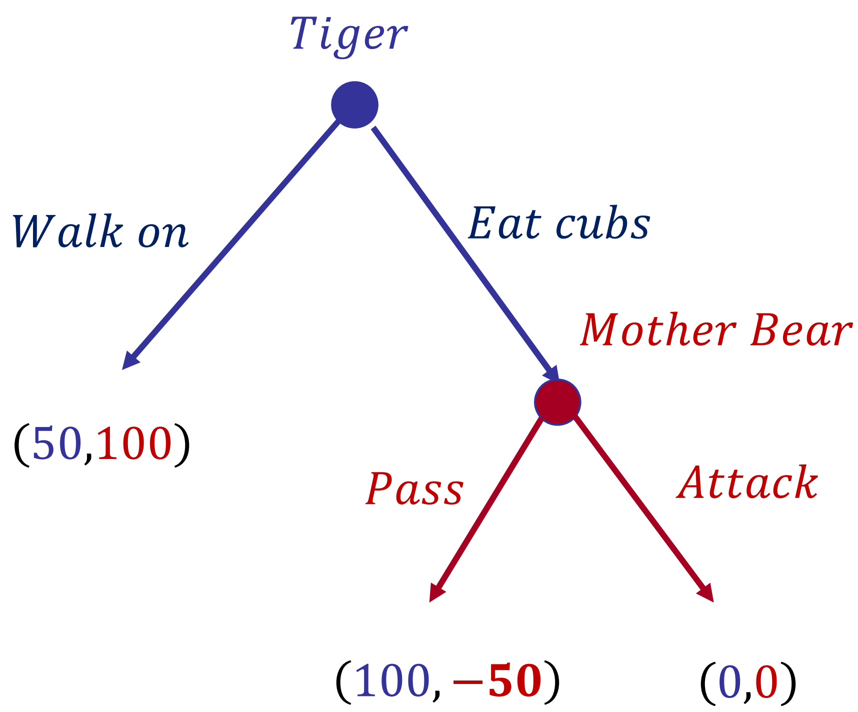
And we can expect evolutionary processes to lead to this one

Perfectness in the strategic form

		Mother Bear	
		Pass	*Attack*
Tiger	*Eat cubs*	100 , 50	0 , 0
	Walk on	50 , 100	50 , 100

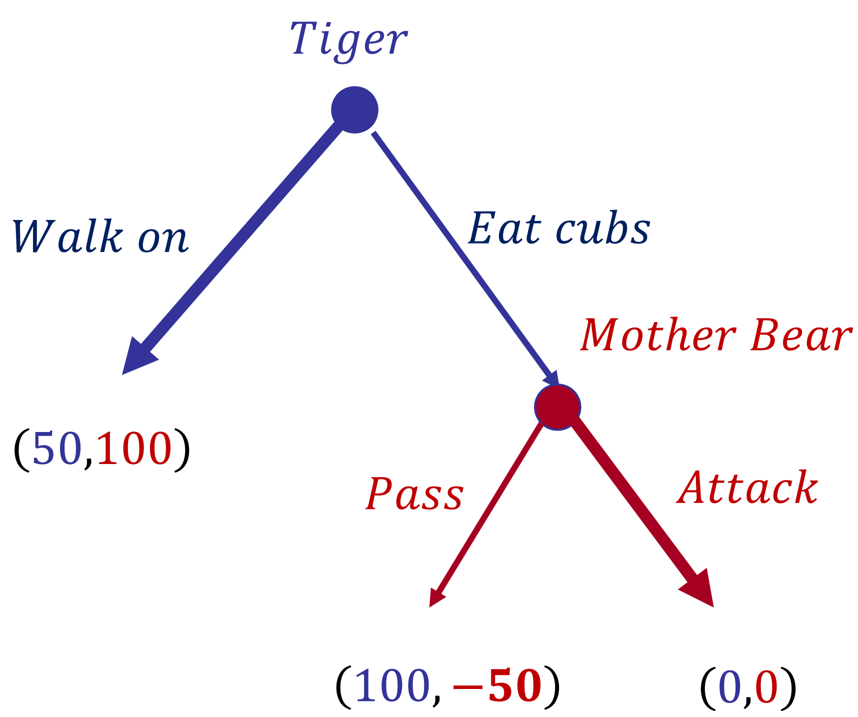
Only the first is trembling-hand perfect

Let’s build vengeance into the payoffs



Recall that the payoffs are supposed to describe behavior

Equilibria in the "emotional" game



Only one equilibrium (in pure strategies), which is also Subgame Perfect

Similarly in the strategic form

		Mother Bear	
		Pass	*Attack*
Tiger	*Eat cubs*	$100 , \mathbf{-50}$	$0 , 0$
	Walk on	$50 , 100$	$50 , 100$

The unique pure strategy equilibrium is trembling-hand perfect

Back to the Ultimatum Game

- Having said all that, emotional payoffs should not be overstated
- In the Ultimatum Game, if the payoffs were in millions of dollars rather than dollars, acceptance of low offers would likely to be higher
- As well as when Player II has to wait before responding

The Ultimatum Game with delay

Let me sleep on it: Delay reduces rejection rates in ultimatum games

Veronika Grimm, Friederike Mengel

Economics Letters, Vol. 111, No. 2 (2011) pp. 113-115

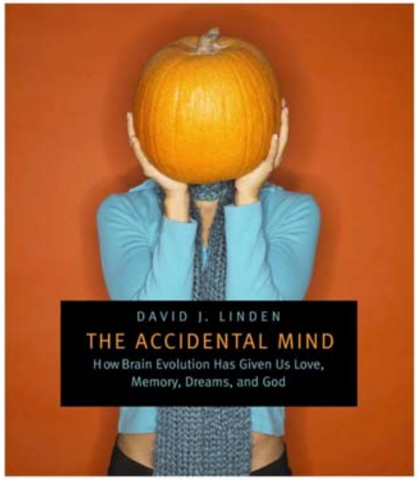
Abstract

Delaying acceptance decisions in the Ultimatum Game drastically increases acceptance of low offers. While in treatments without delay less than 20% of low offers are accepted, 60-80% are accepted as we delay the acceptance decision by around 10. min.

The evolutionary role of emotions

- Not only love for children or partners
- Anger, vengeance can be useful, too
  - Repeated game strategies often use a “punishment phase”
- How about envy?
  - If someone else has more, maybe I missed something?
  - … and our reproduction technology has competition built into it

Can we explain falling in love?



The Accidental Mind (2007)

David Linden (b. 1961)

One common explanation: signaling of future commitment

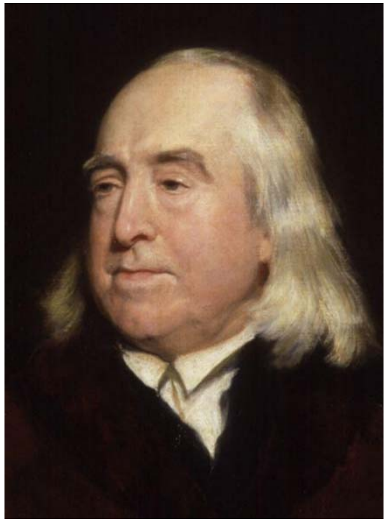
Which might be helpful to get a mate

What cannot be explained?

- You might think that anything can be explained this way
- Indeed, it is not clear that evolution is a testable theory
- But it certainly is a wonderful conceptual framework
- Allowing us to put arguments in order, test the logic of reasoning etc.
- Often people say the same about game theory

Utilitarianism

"It is the greatest happiness of the greatest number that is the measure of right and wrong"



Jeremy Bentham (1748-1832)

Utilitarianism – in formal models

An alternative x is evaluated by

$$U(x) = \sum_{i=1}^{n} u_i(x)$$

where u_i is the utility of individual i

Utilitarianism may favor equality

Say, if $x = (x_1, \dots, x_n)$ is a division of an amount of money

$$U(x) = \sum_{i=1}^{n} u_i(x) = \sum_{i=1}^{n} u_i(x_i)$$

where u_i is the utility of individual i

and x_i is the amount of money she gets

Utilitarianism with a concave utility

$$Max_{x=(x_1,\dots,x_n)} U(x) = \sum_{i=1}^{n} u_i(x_i)$$

$$s.t. \quad \sum_{i=1}^{n} x_i = M$$

if $u_i = u$ and it is concave, the optimal x_i's will be equal

Maximizing the weighted sum of a concave utility

$$Max_{x=(x_1,\ldots,x_n)} \sum_{i=1}^{n} \alpha_i u(x_i)$$

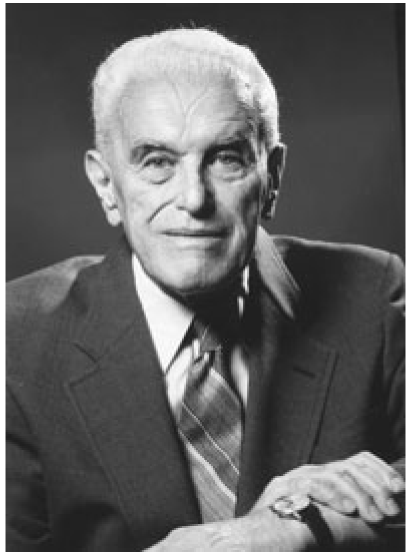
Leads to

- Convex preferences
- Smoothing consumption over time
- Risk aversion
- Egalitarianism

Indeed

Harsanyi suggested to base utilitarianism on decision making under risk

The Impartial Observer – someone who seems to be taken out of society, not knowing who she will be



John Harsanyi (1920-2000)

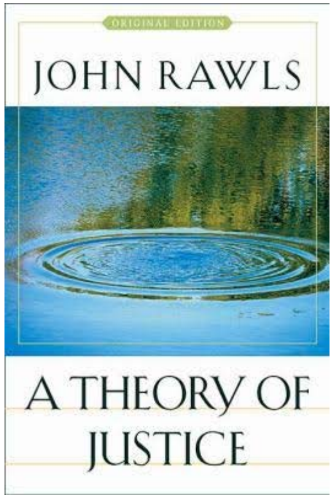
Harsanyi’s mental exercise

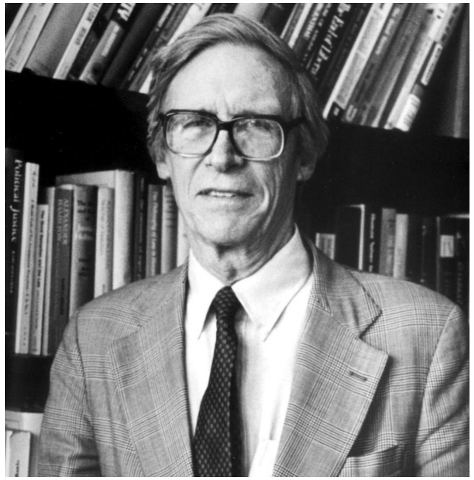
Both in

- (one of) his derivation(s) of utilitarianism from expected utility
- his contribution to incomplete information games

he performed the same mental exercise: going back before we were born, and thinking (in a very Bayesian way) into which world we are going to be born

The same idea as in





John Rawls (1921-2002)

Behind the Veil of Ignorance

In A Theory of Justice (1971)

Kant never liked utilitarianism

The Categorical Imperative:

“Act only according to

that maxim whereby you can, at

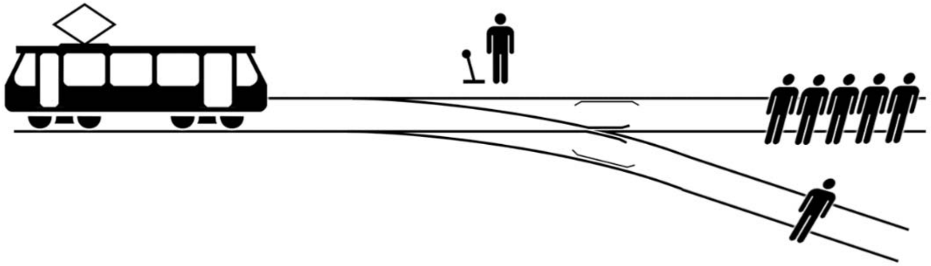
the same time, will that it should

become a universal law”

Immanuel Kant (1724-1804)

Common critiques of utilitarianism

It can justify the killing of innocents



Philippa Foot (1920-2010)

Would you divert the trolley?

Critiques of utilitarianism

- Killing innocents seems wrong

    But don’t we often make such calculations?

- It turns out that the trolley problems invokes different reactions if an “unrelated person” has to be sacrificed (than in the original version)

    Don’t these examples involve many issues of incomplete information and incentives?

In any event

Economists find utilitarianism easy to relate to

- It is consequentialist
- It uses the concept of utility
- And it avoids intangibles like “general rules” and “intentions”

However,

- Even if one accepts utilitarianism, it can be refined
- Do we want other-dependent-preferences to be taken into account?
- Say, envy, schadenfreude…

John Stuart Mill (1806-1873)

For example

- Suppose I fall and break my leg
- You can’t help being amused
- If there are many of you, will this be justified by utilitarianism?
- Would you pay 50 cents for the show?
- Is this moral?

(Is this very different from reality shows?)

Problems with utilitarianism

- Could involve other-dependent preferences
- Requires interpersonal comparison of utility
  - Related to the Other Minds Problem
- Raises problems of elicitation and truth telling

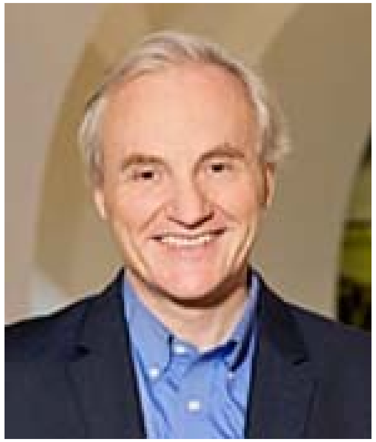
A way out

All of these could be avoided if we impose a putative utility function

(… rather than use the individuals' actual functions)

– more or less what we do in progressive tax policy

Other-dependent preferences



Ernst Fehr (b. 1956)

Klaus M. Schmidt (b. 1961)

Fehr and Schmidt (1999) argued that

there is inequity aversion

Reference

A theory of fairness, competition, and cooperation

Ernst Fehr, Klaus M. Schmidt

Quarterly Journal of Economics, Vol. 114, No. 3 (Aug., 1999), pp. 817-868

Abstract

There is strong evidence that people exploit their bargaining power in competitive markets but not in bilateral bargaining situations. There is also strong evidence that people exploit free-riding opportunities in voluntary cooperation games. Yet, when they are given the opportunity to punish free riders, stable cooperation is maintained, although punishment is costly for those who punish. This paper asks whether there is a simple common principle that can explain this puzzling evidence. We show that if some people care about equity the puzzles can be resolved. It turns out that the economic environment determines whether the fair types or the selfish types dominate equilibrium behavior.

Debates

- Do people dislike inequity also when they are ahead?
- Is the model of “homo economicus” unrealistic because people are nicer, and more altruistic than economic models assume?
- How much negative affect has economics ignores (pride, envy…) ?

Do we have free will?

- Well, most of us say that we sometimes feel like we make choices
- And sometimes this happens when we and others can predict the choice
- Please take five seconds to decide if you want to come up and slap me on the face

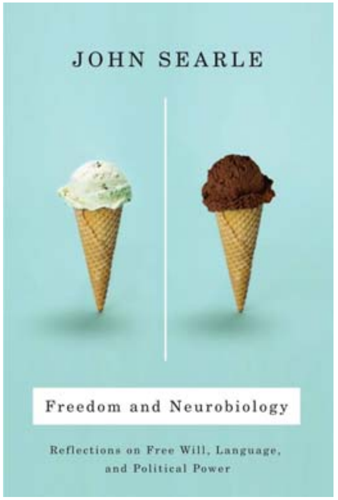
Determinism

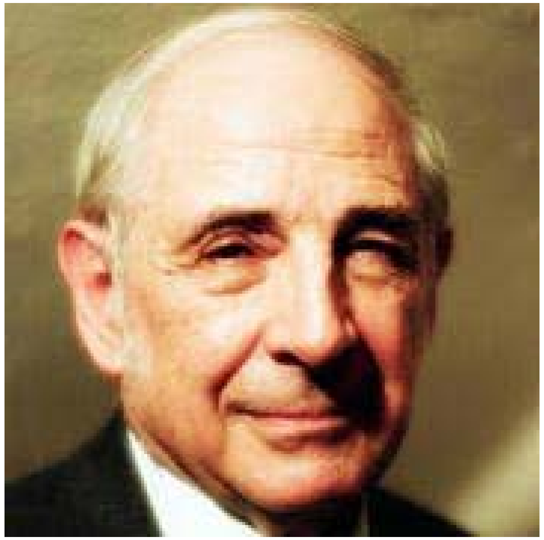
Much of the discussion is about determinism

Material – everything is predetermined by physics

Theological – everything is predetermined, or at least known by God

Determinism and Free Will





John Searle (b. 1932)

Says that he may have to give up the belief in free will, but not quite yet

Determinism and Free Will

Suggested that Free Will might be compatible with physics thanks to Heisenberg's Principle of Uncertainty

Roger Penrose (b. 1931)

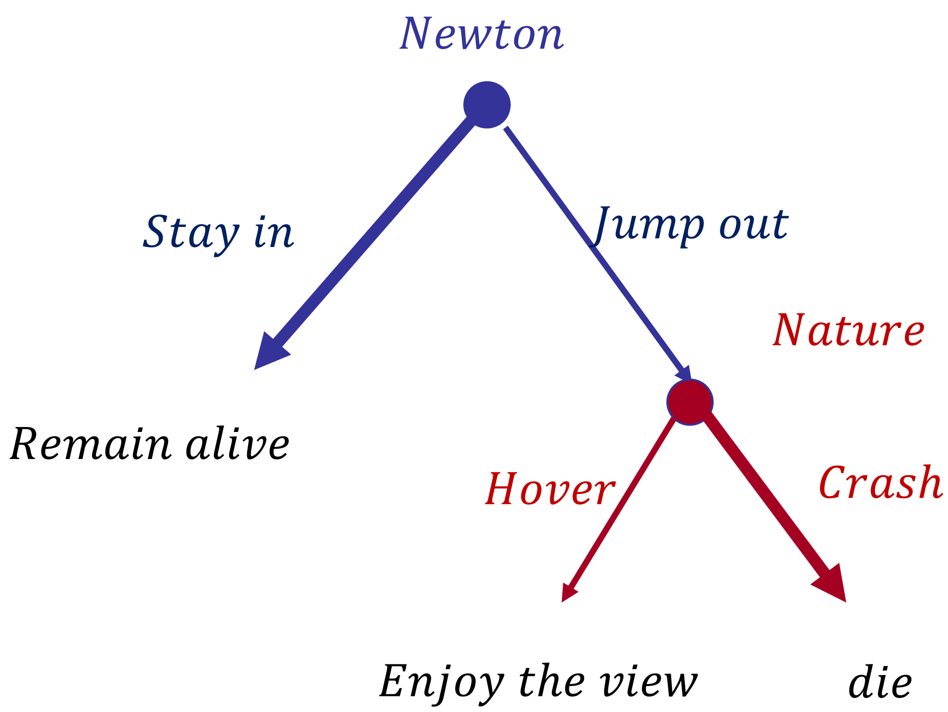
BUT

- … we don’t need to assume that all our choices are known to have a problem
- It suffice to have one
- How can we feel like we’re making a decision when we (and often others) can very well predict it?

A rational illusion

- Imagine you program a robot to make decisions
- You want the robot to learn about its environment, and about itself
- But you want it to imagine more than one world when making a choice
- Sometimes the robot will have to suspend its knowledge/belief about its upcoming choice

Newton by the window



If Newton crosses out *Jump out* the same way he crosses out *Hover*, where’s the decision?

A rational Newton

- Will have to learn about the environment and use this knowledge to form beliefs
- Will have to keep whatever he (and others) learned about his current decision on hold
- He can know a lot about his past and future selves, but not about the self that makes this very decision
- Ignoring what one knows about one's decision is necessitated by rationality

Common Knowledge of Rationality

A riddle

In a certain village there are married couples. The rules are:

- Should a woman know for sure that her husband is unfaithful, she must shoot him on the first night, but she mustn't otherwise
- If a husband is unfaithful, all women know it apart from his wife

One day a traveler comes to visit, gathers all in the main square and says, "There are unfaithful husbands in this village" – and leaves.

The first two nights nothing happens, but on the third night shots are heard. How many shots were there?

Solution

Three shots – and we can prove by induction, that, for every $k \geq 1$, if there are k unfaithful husbands, they all get shot on the k-th night.

Start with $k = 1$

If there is only one, his wife doesn't know that there exist any

When she hears that there is (at least one), she figures out it must be hers.

Next, $k = 2$

If there are two, each of their wives knows about the phenomenon. It doesn't have to be her husband, there's another one she knows about.

But… if her husband were faithful, the other one would have been the only one $(k = 1)$ … And should have gotten shot the first night.

Solution – cont.

Next, $k = 3$

Each of their wives knows about the phenomenon. She actually knows about two others.

But… if her husband were faithful, the other two would have been the only ones $(k = 2)$. And should have gotten shot the second night.

Wait a minute, what information did the traveler give the wives?

After all, they all knew that there are unfaithful husbands (each knew about 2 or 3) – what was new?

Common Knowledge

An event A is common knowledge among a set of agents, if

- Everyone knows A
- Everyone knows that everyone knows A
- Everyone knows that everyone knows that everyone knows A
- …

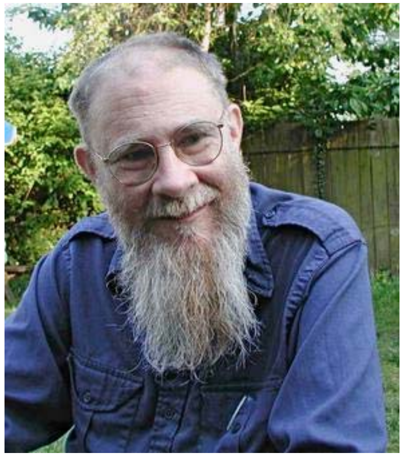
Common Knowledge

The concept appeared in

- Philosophy
- Game theory
- Computer Science

Common Knowledge in philosophy

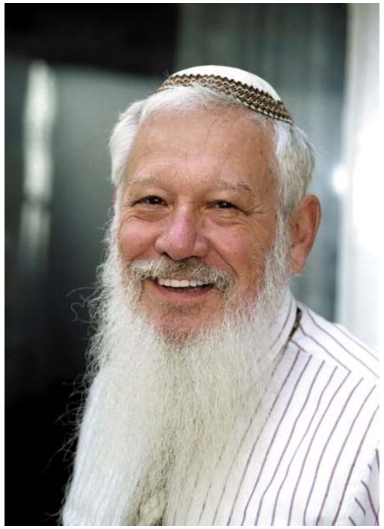
Convention: A Philosophical Study (1969)



David K Lewis (1941-2001)

Common Knowledge in game theory

Defined common knowledge by a partition model



Robert J. Aumann (b. 1930)

Common Knowledge in computer science

Joseph Y. Halpern (b. 1953)

Yoram Moses

The partition model

Each player has a partition of the state space, such that, at each state, she knows that the state is in the event (in her partition) containing the state (but not more than that)

Say,

$$S = \{1,2,3,4\}$$

$$P = \{\{1,2\},\{3,4\}\}$$

Why a partition?

We can think of a relation I between two states, where

sIs' means “at state s the player thinks that s' is possible”

The system S5

Assumptions on knowledge:

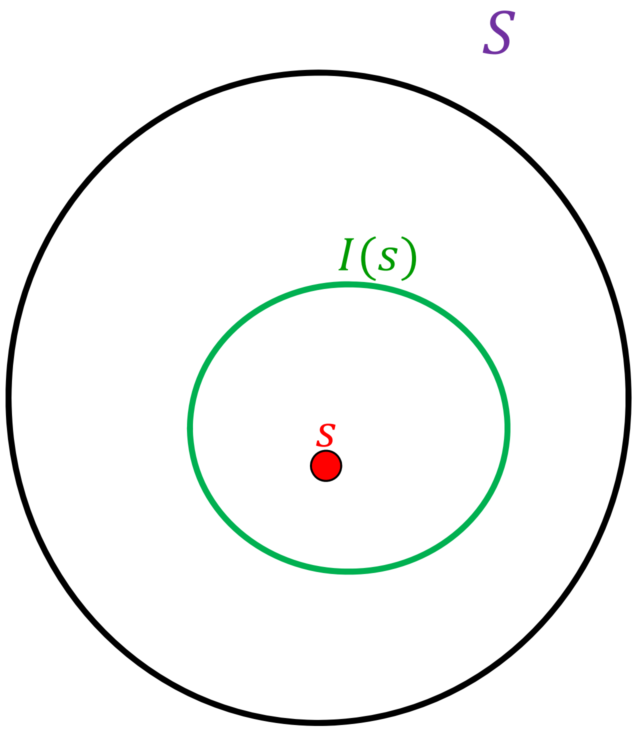
T	(truth)	$kp \to p$
K	(implication)	$k(p \to q) \to (kp \to kq)$
S4	(positive introspection)	$kp \to kkp$
S5	(negative introspection)	$\neg kp \to k(\neg kp)$

S5 implies that I is an equivalence relation

Reflexivity

I is reflexive:

If s is really the case, one cannot know that it isn't (using truth)



Symmetry

I is symmetric: suppose that

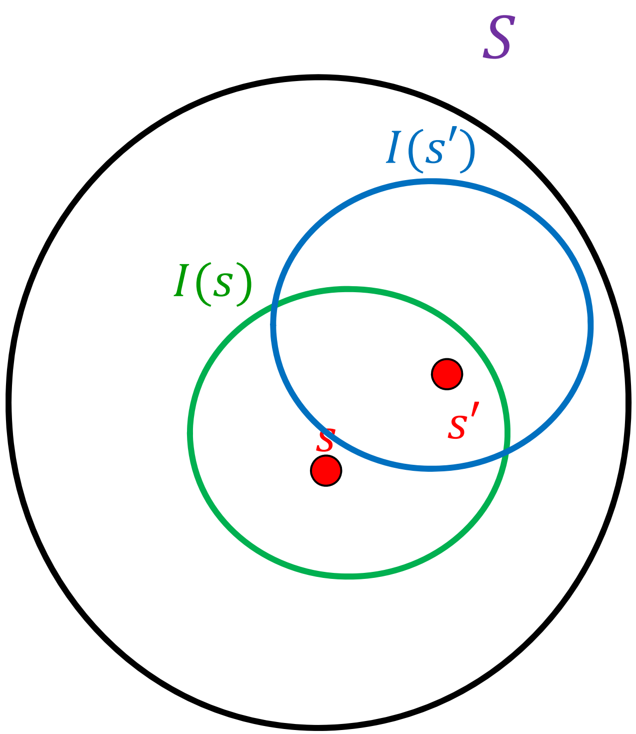
$$s' \in I(s)$$

but not

$$s \in I(s')$$

Then we could say, "if it were s' we'd know that s were impossible, but we don't know that!"

(using negative introspection)



Transitivity

I is transitive: suppose that

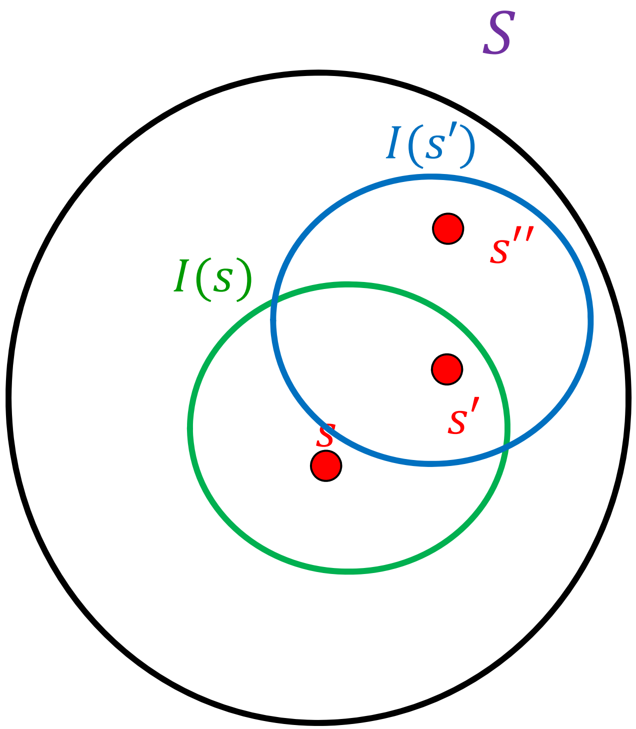
$$s' \in I(s),\ s'' \in I(s'),$$

but not

$$s'' \in I(s)$$

Then we could say, “if it were s' we’d think that s'' were impossible, but we know it isn’t!”

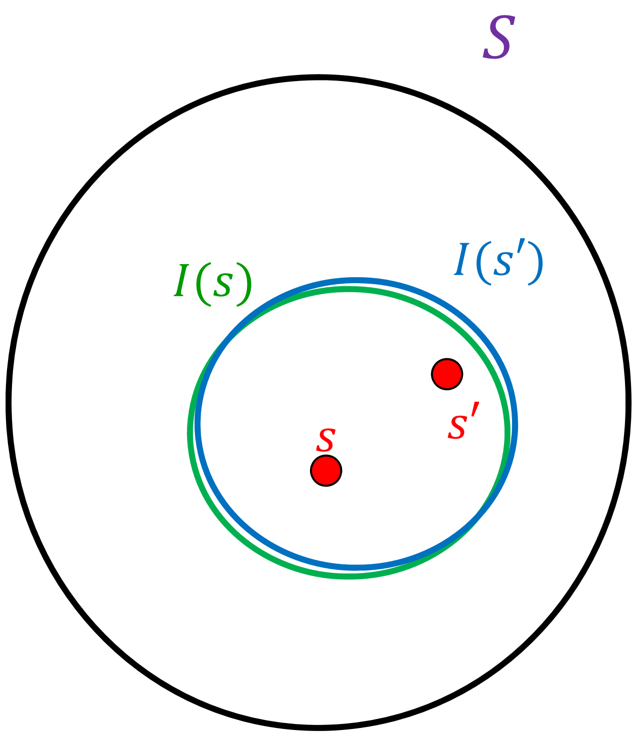
(using positive introspection)



The partition

… is simply the collection of equivalence classes of the relation I

For any two states, s, s', either $I(s), I(s')$ are disjoint, or equal



CK in the partition model

Suppose we have a state space

$$S = \{1,2,3,4\}$$

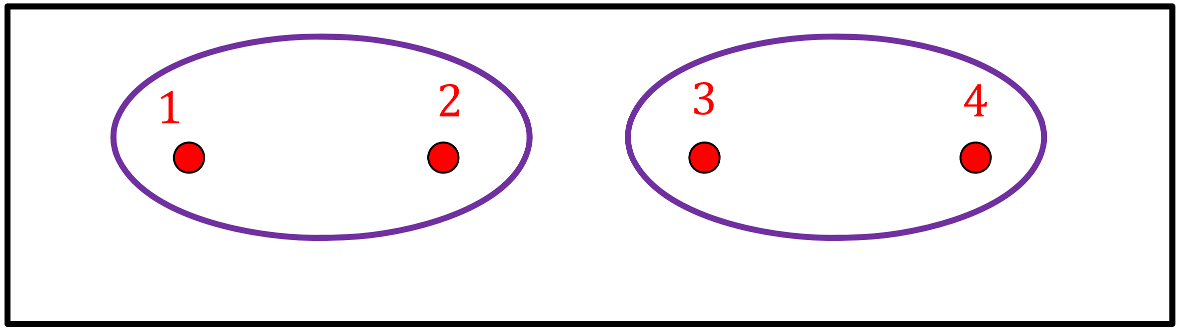
And two players with partitions

$$P_a = \{\{1,2\},\{3,4\}\}$$

$$P_b = \{\{1,3\},\{2,4\}\}$$

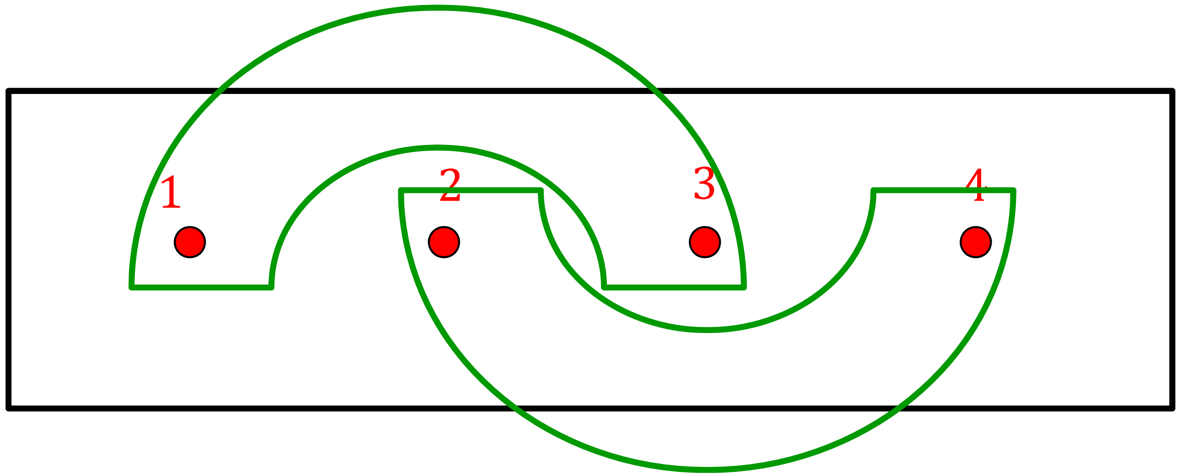
What is common knowledge between them?

Player a's partition



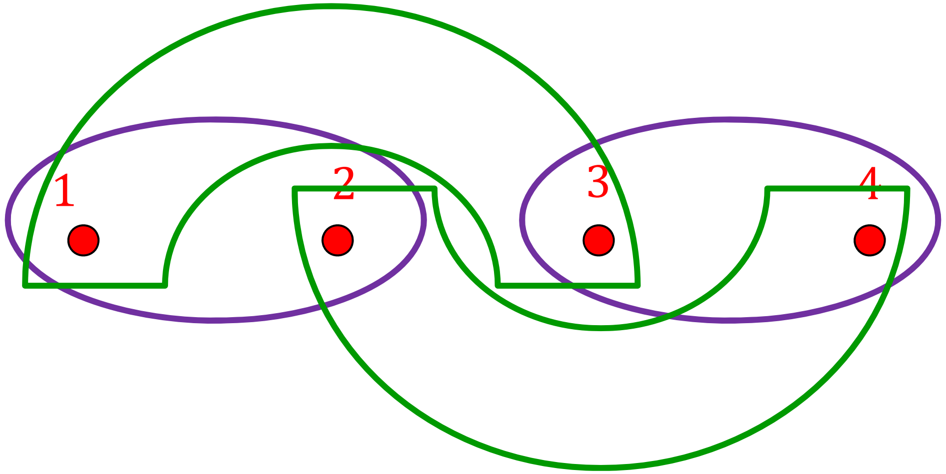
$$P_a = \{\{1,2\},\{3,4\}\}$$

Player b's partition



$$P_b = \left\{\{1,3\},\{2,4\}\right\}$$

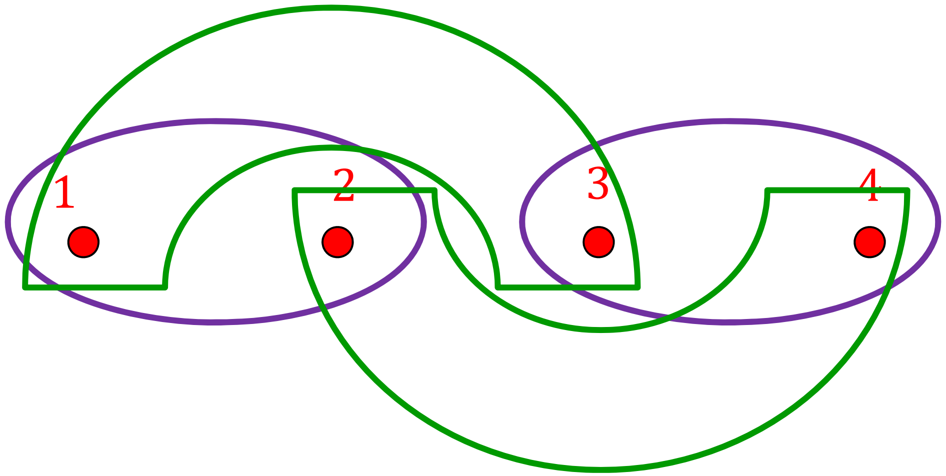
CK in the partition model – cont.



$$P_a = \{\{1,2\},\{3,4\}\}$$

$$P_b = \{\{1,3\},\{2,4\}\}$$

CK in the partition model – cont.



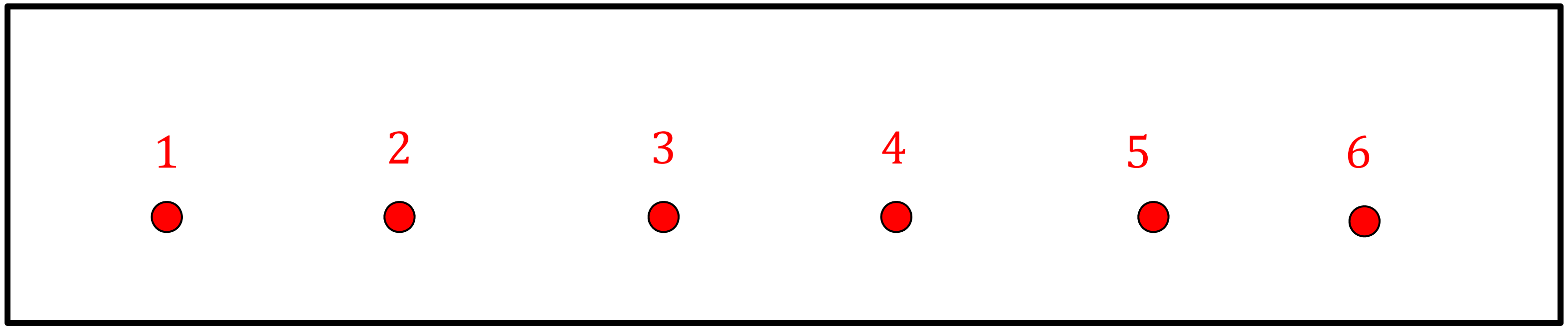
For example, at 1 a knows it's 1 or 2;

And she knows that, if it's 1, then b thinks that 3 is possible

And she knows that, if it's 2, then b thinks that 4 is possible

Only $S = \{1,2,3,4\}$ is CK between the players

But



$$P_a = \{\{1,2\},\{3,4\},\{5,6\}\}$$

$$P_b = \{\{1,3\},\{2,4\},\{5\},\{6\}\}$$

The commonly known event will be one of

$$P_{CK} = \{\{1,2,3,4\},\{5,6\}\}$$

Reference

Agreeing to disagree

Robert J. Aumann

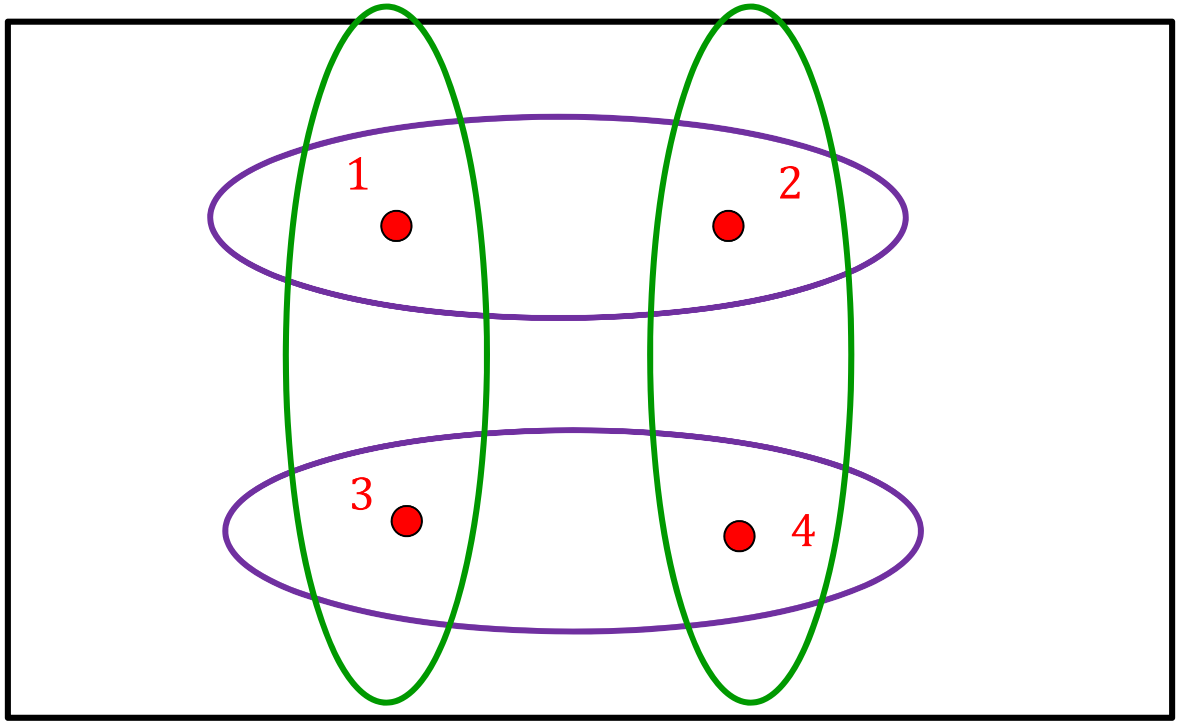
Annals of Statistics, Vol. 4, No. 6 (Nov., 1976), pp. 1236-1239

Abstract

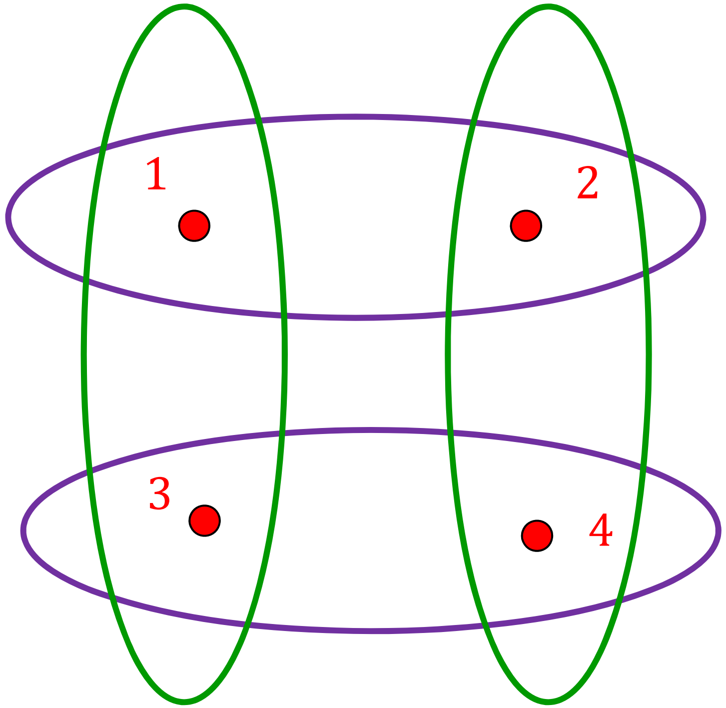
Two people, 1 and 2, are said to have *common knowledge* of an event *E* if both know it, 1 knows that 2 knows it, 2 knows that 1 knows it, 1 knows that 2 knows that 1 knows it, and so on.

Theorem. If two people have the same priors, and their posteriors for an event *A* are common knowledge, then these posteriors are equal.

Agreeing to disagree – logic

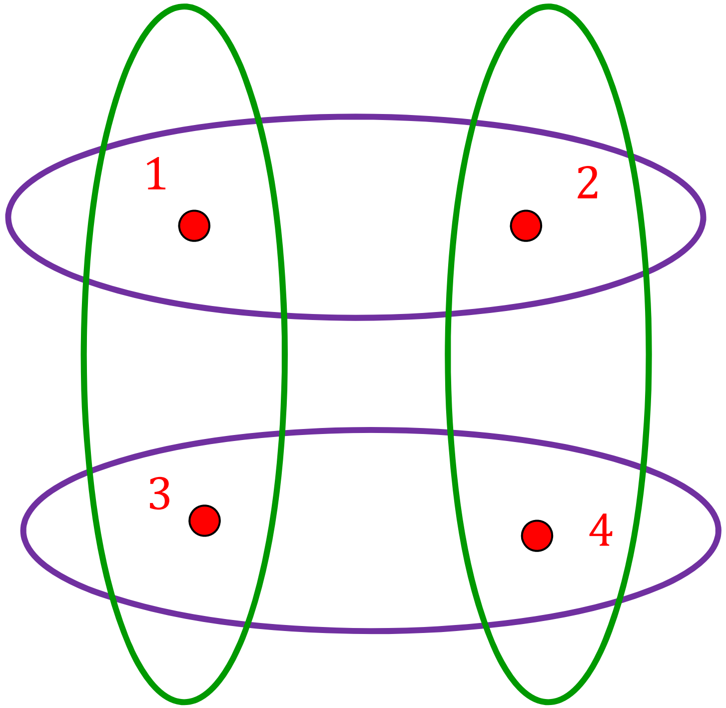


The model is known…



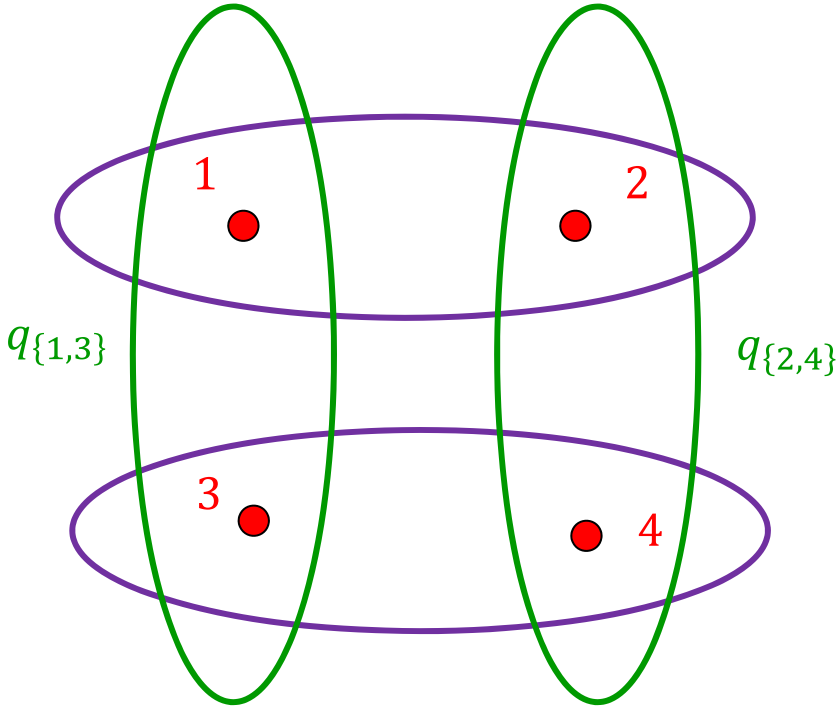
The posterior of each player at each state is known to all. It is "common knowledge".

The CK event



The smallest event that is CK – here all four states

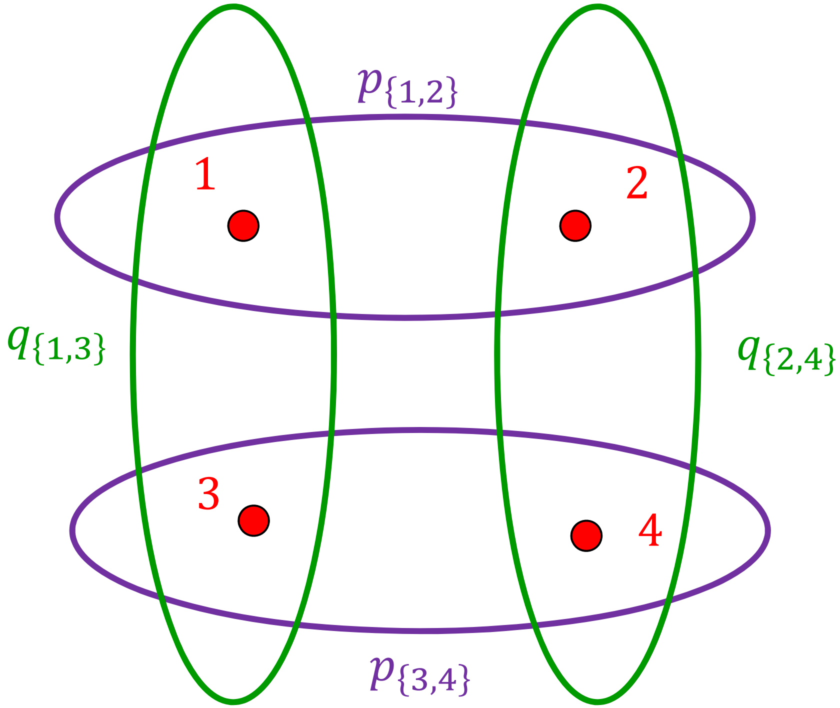
If the posterior is CK



If b's posterior is CK, we must have $q_{\{1,3\}} = q_{\{2,4\}}$

And then they are both equal to b's prior, too!

Important:



Equality of posteriors results from equality of priors:

$$q_{\{1,3\}} = q_{\{2,4\}} = q$$
$$p_{\{1,2\}} = p_{\{3,4\}} = p$$

Following “Agreeing to Disagree”

No-betting results

No-trade results

Intuitively, if someone wants to trade with me, I can infer something about the information they received even if I don’t have direct access to that information

Reference

Information, trade and common knowledge

Paul Milgrom, Nancy Stokey

Journal of Economic Theory, Vol. 26, No. 1 (Feb., 1982), pp. 17-27

Abstract

In any voluntary trading process, if agents have rational expectations, then it is common knowledge among them that the equilibrium trade is feasible and individually rational. This condition is used to show that when risk-averse traders begin at a Pareto optimal allocation (relative to their prior beliefs) and then receive private information (which disturbs the marginal conditions), they can still never agree to any non-null trade. On markets, information is revealed by price changes. An equilibrium with fully revealing price changes always exists, and even at other equilibria the information revealed by price changes "swamps" each trader's private information.

Common knowledge among computers

The Byzantine Generals problem: Two generals need to coordinate an attack. Mis-coordination is a disaster.

One sends a messenger, but the latter might be captured, so the sender awaits a confirmation.

But then the receiver needs to know that the confirmation has arrived. So we need a confirmation of the confirmation.

When will the arrival of the first message be common knowledge?

Reference

The electronic mail game: strategic behavior under “almost common knowledge”

Ariel Rubinstein

American Economic Review, Vol. 79, No. 3 (June, 1989), pp. 385-391

Abstract

The paper addresses a paradoxical game-theoretic example which is closely related to the coordinated attack problem. Two players have to play one of two possible coordination games. Only one of them receives information about the coordination game to be played It is shown that the situation with "almost common knowledge" is very different from when the coordination game played is common knowledge.

Another puzzle

A computer sends a message to another, and it is CK that it takes $60''$ to arrive. If sent at t, at $t+60''$ it will be CK that it has indeed arrived.

Now there's a technological improvement. The message can take any amount of time up to $60''$ seconds to arrive. If sent at t, when will it be CK that the message has indeed arrived?

Solution

Never.

When sending, the first computer has to wait $60''$ to know the message has arrived. (That was the case before the improvement, too.)

The receiver, when getting the message, has to wait $60''$ to know that the sender knows that the message has arrived. (That wasn't the case before!)

So now the sender, at time $t + 60''$ can be sure that the message made it, but it knows that the receiver only now starts counting another $60''$ until it knows that the sender knows.

And on it goes…

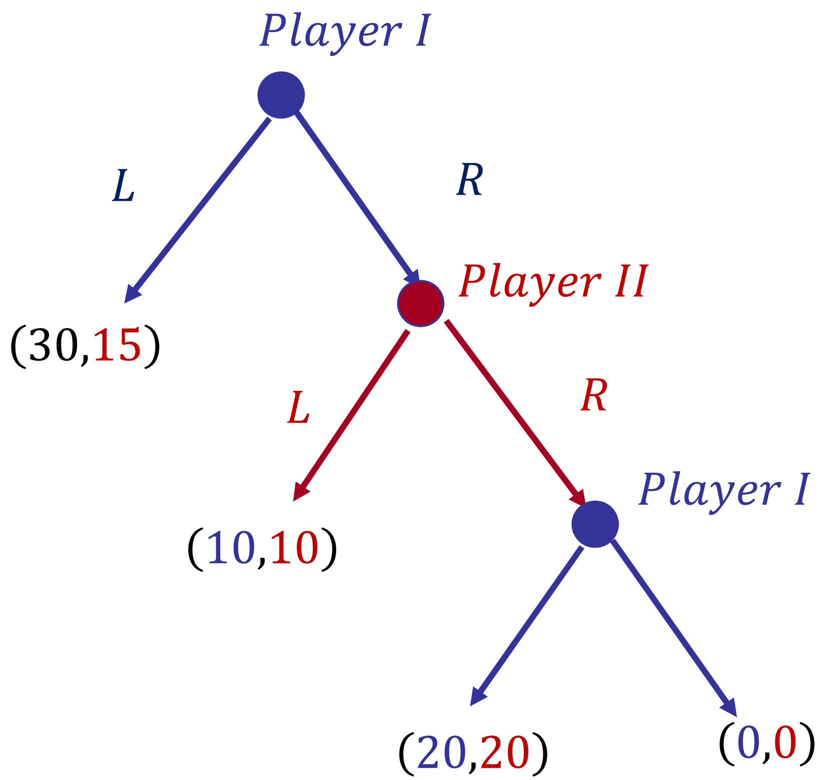
CK of Rationality and Backward Induction (BI)

There are theoretical issues: does CK of rationality imply the BI?

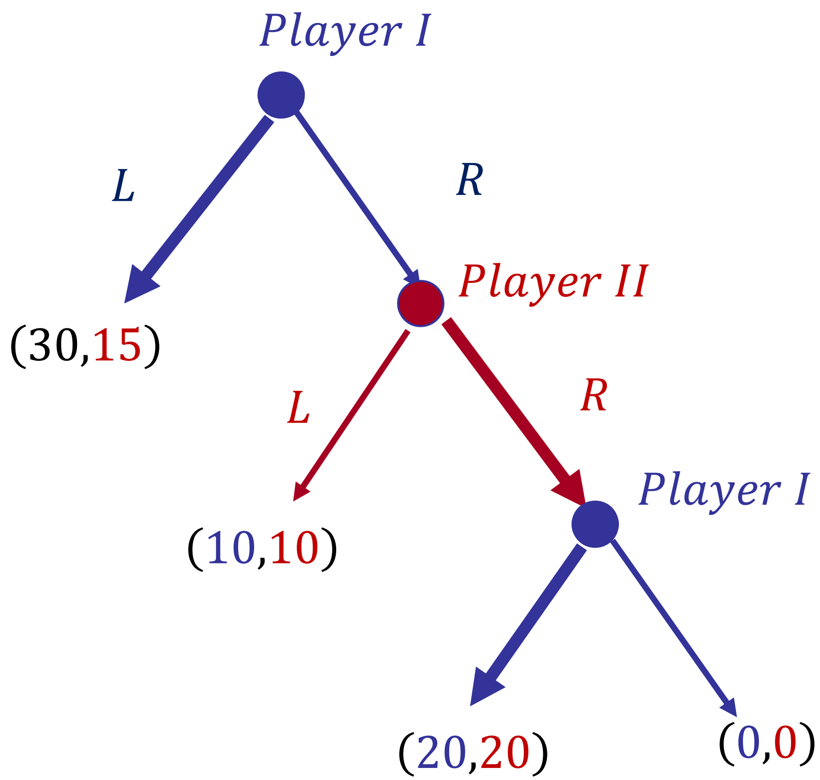
After all, one deviation from the BI suffices to refute the theory

What will happen then?

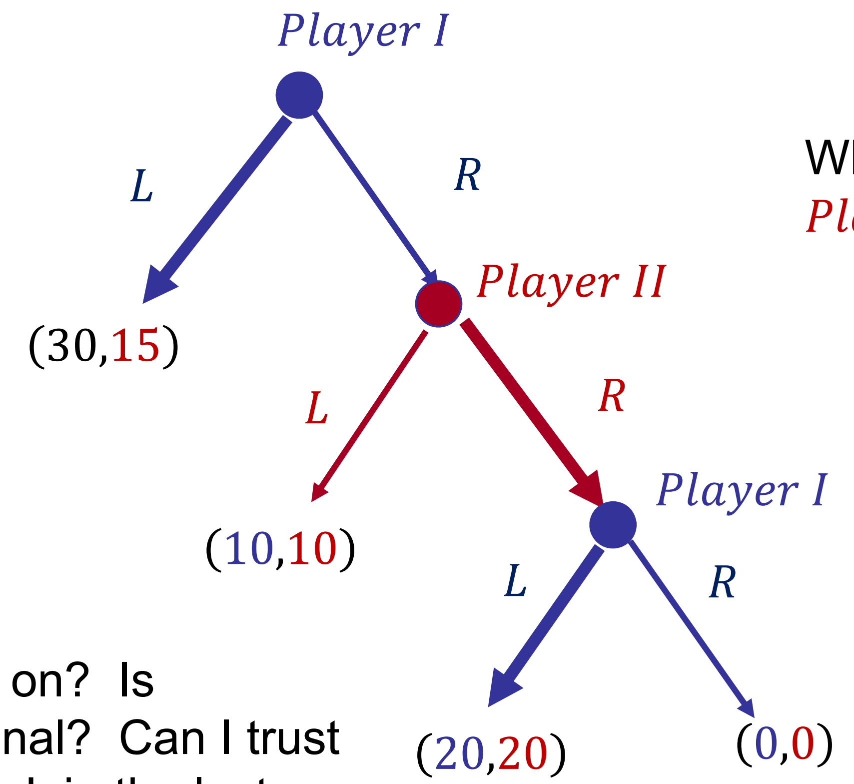
Consider the game



The BI solution



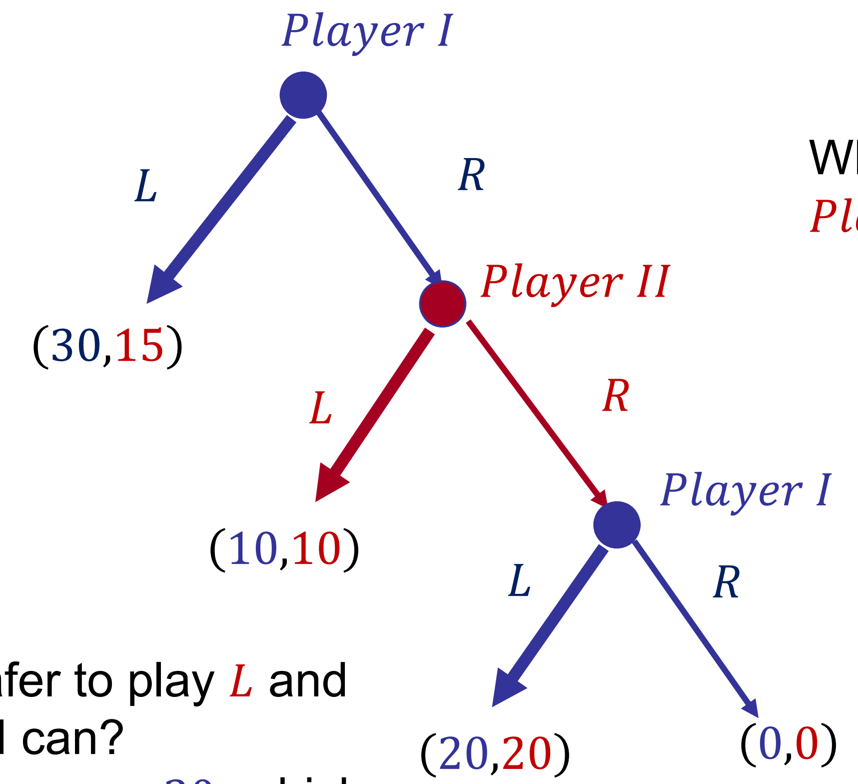
When the BI solution is not followed



What should *Player II* think?

What's going on? Is *Player I* rational? Can I trust her to choose L in the last node, if we get there?

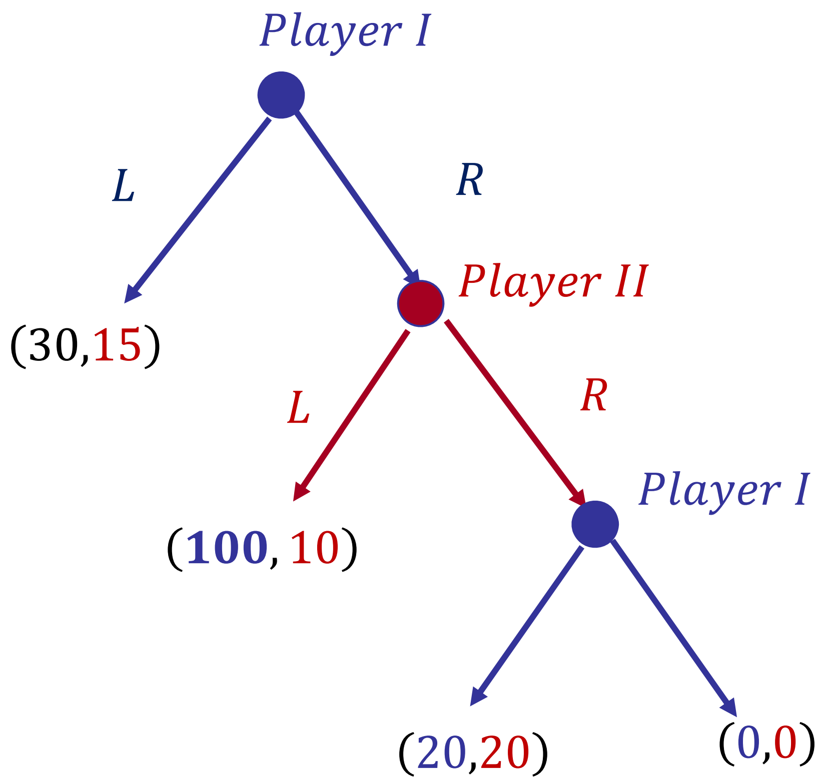
When the BI solution is not followed



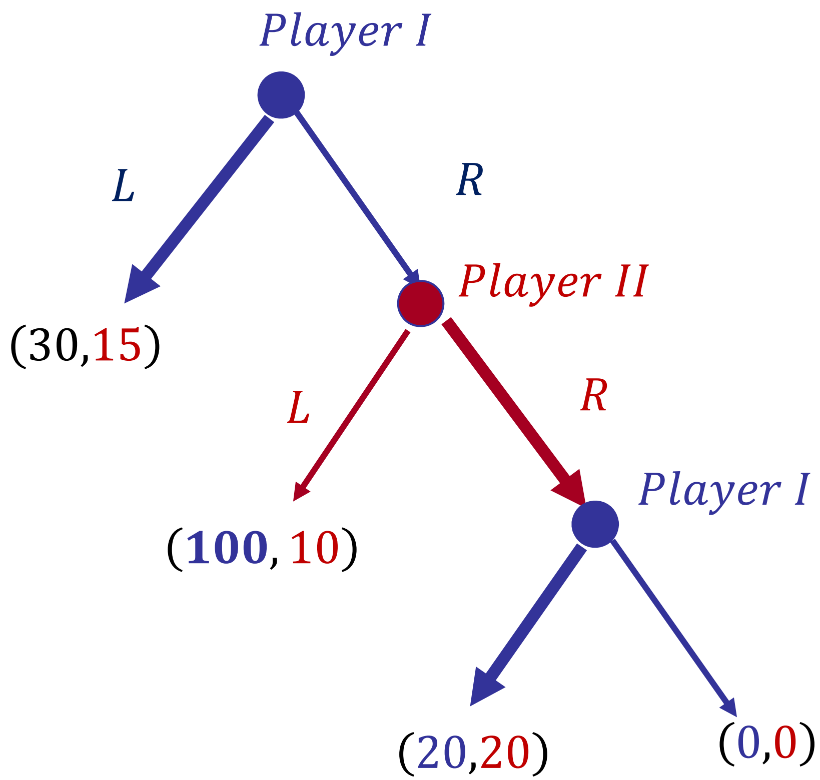
What should $Player\ II$ think?

Maybe it's safer to play L and get 10 while I can?
$Player\ I$ gave up on 30, which is higher than anything in this subgame!

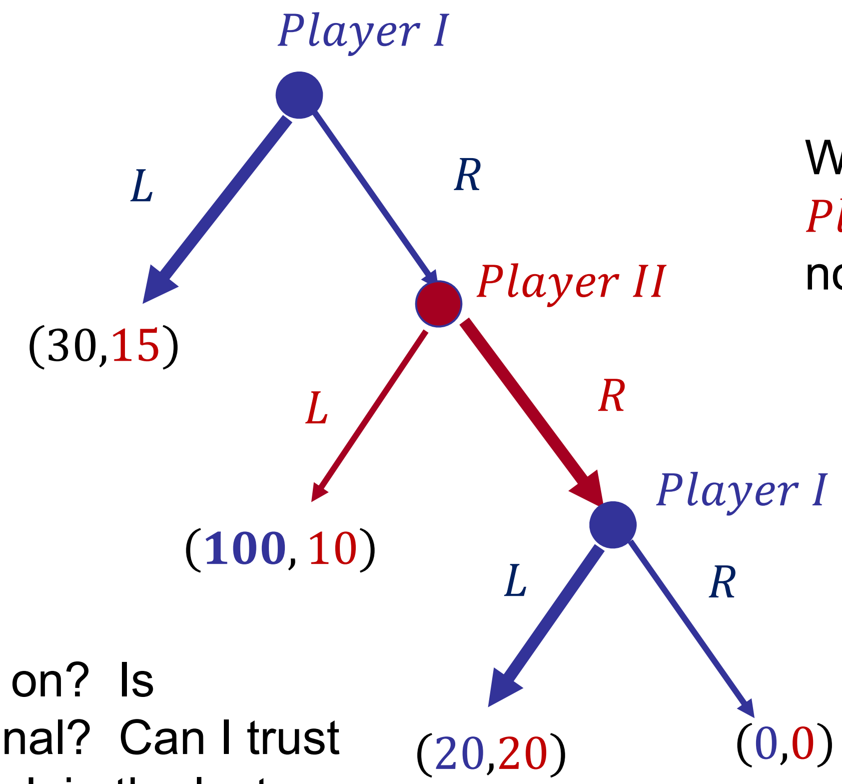
Now let's consider the game



The BI solution is the same



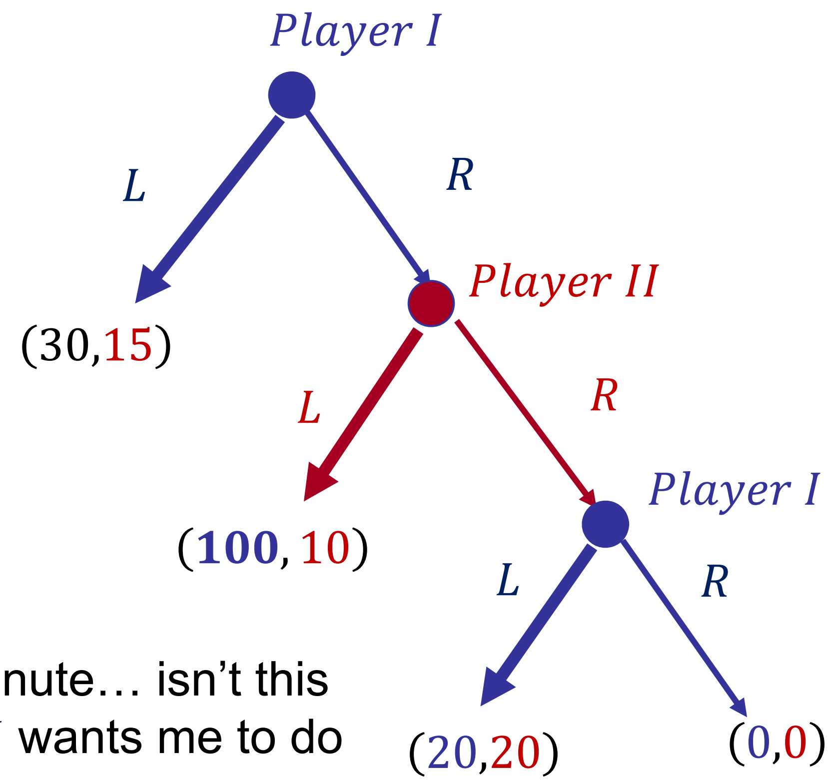
But if the BI solution is not followed



What should *Player II* think now?

What's going on? Is *Player I* rational? Can I trust her to choose L in the last node, if we get there?

Maybe play safe?



But, wait a minute… isn't this what *Player I* wants me to do precisely?

CK of Rationality and BI

Thus there are theoretical questions:

- Does CK of rationality imply the BI solution
- Is CK of rationality logically consistent to begin with?

But there are also the more practical questions,

- Is CK of rationality realistic?
- Is the BI solution a good model?

Selten’s Chain Store Pradox

- A chain store may or may not engage in a price war in each location against a different opponent
- The war pays off only if it builds reputation and deters future opponents
- But the BI says it would never fight
- Selten suggested three levels of decision making:
  - Routine
  - Imagination
  - reasoning

Reference

The chain store paradox

Reinhard Selten

Theory and Decision, Vol. 9 (1978), pp. 127-159

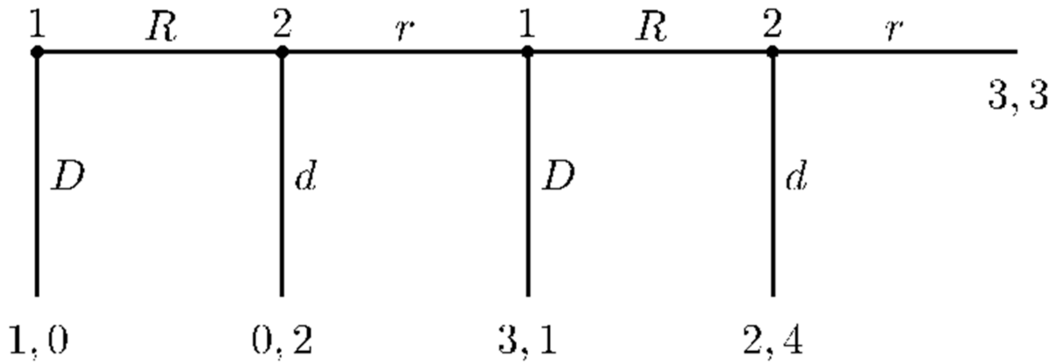
Abstract

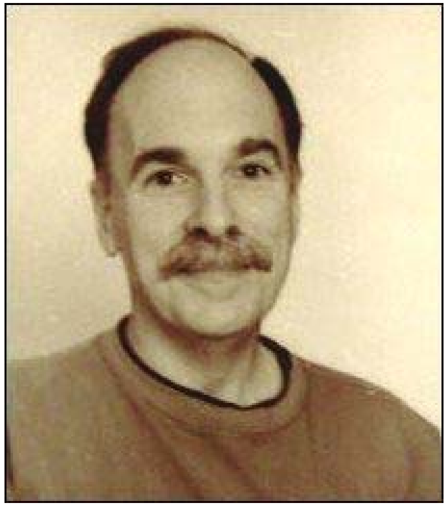
The chain store game is a simple game in extensive form which produces an inconsistency between game theoretical reasoning and plausible human behavior. Well-informed players must be expected to disobey game theoretical recommendations.

The chain store paradox throws new light on the well-known difficulties arising in connection with finite repetitions of the prisoners' dilemma game. Whereas these difficulties can be resolved by the assumption of secondary utilities arising in the course of playing the game, a similar approach to the chain store paradox is less satisfactory.

It is argued that the explanation of the paradox requires a limited rationality view of human decision behavior. For this purpose a three-level theory of decision making is developed, where decisions can be made on different levels of rationality. This theory explains why insight into the rational solution of a decision problem does not necessarily mean that the corresponding course of action will be taken.

Rosenthal's Centipede Game





Robert W. Rosenthal
(1945-2002)

The payoff to each player increases every two periods, but if the other player won't play Right/right, she better grab the money and run

… and imagine it's much longer

Reference

Games of perfect information, predatory pricing and the Chain-Store Paradox

Robert W. Rosenthal

Journal of Economic Theory Vol. 25 (1981), pp. 92-100

(No abstract)

The Prisoner's Dilemma

		Player II	
		C	D
Player I	C	$3 , 3$	$0 , 4$
	D	$4 , 0$	$1 , 1$

(D, D) isn't just the unique Nash equilibrium

– the strategy D is dominant for each

The story behind the PD

- The story about “betrayal” causes confusion
- Recall that the utility numbers should reflect behavior
- $4 > 3$ means that, given the choice, the former will be picked
- All payoffs (psychological etc.) are incorporated into these numbers

The PD again

		Player II	
		C	D
Player I	C	3 , 3	0 , 4
	D	4 , 0	1 , 1

So it may be better to think of a game in which each can get $3,000 transferred to the other (from the bank) or $1,000 to oneself

The point of the PD

- Some social interactions are not conducive to cooperation

    because the only equilibria are not Pareto optimal

- So we may need to change the rules of the game

Changing payoffs in the PD

		Player II	
		C	D
Player I	C	3 , 3	0 , **0**
	D	**0** , 0	1 , 1

Now (C, C) will (also) be an equilibrium

How do we change payoffs?

- Laws
- Social norms
- Guilt induction

And repeating the interaction can also help

Indeed, small communities can foster cooperation

A problem with repetition

- If the game is repeated indefinitely (say, fixed probability of continuation at any stage) – fine
- If it is played only T times and T is CK – the only equilibrium is no cooperation throughout
  - Proof by induction
- Luckily, people may not think that far…

Bounded rationality can help

- If people have an implicit belief in an infinite horizon, that's not so bad
- This is why money has value
- But what happens if they start thinking?
  - Or learn from experience?

Finitely repeated PD

- In experiments people often cooperate quite a bit
- Until a few stages before the end
- But then, if the entire repeated game is repeated, they may learn not to cooperate

Level-K reasoning

- An idea: classify players by the number of “layers” of reasoning they engage in
- For example (a “Keynes Beauty Contest”) players have to guess a number in $[0,100]$ and be as close as possible to $\frac{2}{3}$ of the average of all the others
- The only equilibrium is to guess zero, but the distribution of answers gives an idea about K

Reference

Evolution of Smart_n Players

Dale O. Stahl

Games and Economic Behavior Vol. 5 No. 4 (1993), pp. 604-617

Abstract

To model the evolution of strategic intelligence, player types are drawn from a hierarchy of "smartness" analogous to the levels of iterated rationalizability. Nonrationalizable strategies die out, but when higher levels of smartness incur maintenance costs, being right is always as good as being smart. Moreover, if a manifest way to play emerges, then dumb players never die out, while smarter players with positive maintenance costs vanish. These results call to question the standard game-theoretic assumption of super-intelligent players.

Reference

Experimental evidence on players' models of other players

Dale O. Stahl, Paul Wilson

Journal of Economic Behavior and Organization Vol. 25 No. 3 (1994), pp. 309-327

Abstract

We pose a hierarchial model of strategic thinking and conduct an experiment to test this theory as well as other solution concepts for symmetric (3 × 3) games. A level-0 type plays unpredictably, a level-1 type acts as if everyone else were level-0 types, and a level-2 type acts as if all other players were level-0 and level-1 types. In a model with level-0,…, level-2, and Nash types, we estimated that an insignificant portion of the participants were level-0 types, 24% were level-1 types, 49% were level-2 types, and the remaining 27% were Nash types.

Reference

Unraveling in guessing games: An experimental study

Rosemarie Nagel

American Economic Review Vol. 85 No. 5 (1995), pp. 1313-1326

Abstract

The structure of a particular game is found to be conducive for the investigation of whether and how the mental process of a player considers other players' conscious-reasoning behavior. In this game, a large number of players are required to concurrently state number from 0 to 100. The winner is the contestant with a chosen number that is closest to the product of the mean of all numbers stated and a parameter p, which is a commonly known predetermined positive parameter of the game. The game is played four times.

Well Being

Problem

Mary’s direct boss just quit, and you’re looking for someone for the job. You don’t think that Mary is perfect for it. By contrast, Jane seems a great fit. But it may be awkward to promote Jane and make Mary her subordinate. A colleague suggested that you go ahead and do this, but give both of them a nice raise to solve the problem.

Money and Well-Being

Money isn't everything

- Low correlation between income and well-being
- The relative income hypothesis (Duesenberry, 1949)
- Higher correlation within a cohort than across time (Easterlin, 1974)
- Relative to an aspiration level

Other determinants of Well-Being

- Love, friendship
- Social status
- Self fulfillment
  - So how should we measure “success”?

Subjective Well Being

- We already mentioned Helson and Adaptation Level Theory
- Some of his disciples applied it to well-being
- Brickman and Coates (1971): the hedonic treadmill
- Brickman, P., Coates, D., & Janoff-Bulman, R. (1978). Lottery winners and accident victims: Is happiness relative? *Journal of Personality and Social Psychology, 36*(8), 917–927. https://doi.org/10.1037/0022-3514.36.8.917
- https://www.ted.com/talks/dan_gilbert_asks_why_are_we_happy/footnotes?referrer=playlist-324

Reference

Lottery winners and accident victims: Is happiness relative?

Philip Brickman, Dan Coates, Ronnie Janoff-Bulman

Journal of Personality and Social Psychology, Vol. 36 No. 8 (1978), pp. 917-927

Abstract

Adaptation level theory suggests that both contrast and habituation will operate to prevent the winning of a fortune from elevating happiness as much as might be expected. Contrast with the peak experience of winning should lessen the impact of ordinary pleasures, while habituation should eventually reduce the value of new pleasures made possible by winning. Study 1 compared a sample of 22 major lottery winners with 22 controls and also with a group of 29 paralyzed accident victims who had been previously interviewed. As predicted, lottery winners were not happier than controls and took significantly less pleasure from a series of mundane events. Study 2, using 86 Ss who lived close to past lottery winners, indicated that these effects were not due to preexisting differences between people who buy or do not buy lottery tickets or between interviews that made or did not make the lottery salient. Paraplegics also demonstrated a contrast effect, not by enhancing minor pleasures but by idealizing their past, which did not help their present happiness.

Subjective Well Being – Methodology

- Strack, Martin, and Schwarz (1988) showed that the order of questions can change correlation of factual ones with assessment of well-being

- Schwarz and Clore (1983) showed that people could "deduct" the effect of weather in responding to well-being questions, provided they were induced to notice it

Reference

Priming and communication: Social determinants of information use in judgments of life satisfaction

Fritz Strack, Leonard L. Martin, Norbert Schwartz

European Journal of Social Psychology, Vol. 18 No. 5 (1988), pp. 429-442

Abstract

Two experiments examined the effects of answering a question about a specific component of life satisfaction on respondents' assessment of their overall satisfaction with life. The results suggest that the use of primed information in forming subsequent judgments is determined by Grice's conversational norms. In general, answering the specific question increases the accessibility of information relevant to that question. However, the effect that this has on the general judgment depends on the way in which the two questions are presented. When the two questions are merely placed in sequence without a conversational context, the answer to the subsequent general question is based in part on the primed specific information. As a result, the answer to the general question becomes similar to that for the specific question (i.e. assimilation). However, this does not occur when the two questions are placed in a communication context…

Reference

Mood, misattribution, and judgments of well-being: Informative and directive functions of affective states

Norbert Schwartz, Gerald Clore

Journal of Personality and Social Psychology, Vol. 45 No. 3 (Sep. 1983), pp. 513-523

Abstract

Investigated, in 2 experiments, whether judgments of happiness and satisfaction with one's life are influenced by mood at the time of judgment. In Exp I, moods were induced by asking 61 undergraduates for vivid descriptions of a recent happy or sad event in their lives. In Exp II, moods were induced by interviewing 84 participants on sunny or rainy days. In both experiments, Ss reported more happiness and satisfaction with their life as a whole when in a good mood than when in a bad mood. However, the negative impact of bad moods was eliminated when Ss were induced to attribute their present feelings to transient external sources irrelevant to the evaluation of their lives; but Ss who were in a good mood were not affected by misattribution manipulations. The data suggest that (a) people use their momentary affective states in making judgments of how happy and satisfied they are with their lives in general and (b) people in unpleasant affective states are more likely to search for and use information to explain their state than are people in pleasant affective states.

More on method

- And will the lottery winners be willing to swap?
- What about having children?

The Day Reconstruction Method

Kahneman and colleagues: memories of experiences are not to be trusted

We need something more objective

Ask people what they did and how long, ask others how much fun it is

Reference

A Survey Method for Characterizing Daily Life Experience: The Day Reconstruction Method

Daniel Kahneman, Alan B. Krueger, David A. Schkade, Norbert Schwarz, Arthur A. Stone

Science, Vol. 306 No. 5702 (Dec. 2004), pp. 1776-1780

Abstract

The Day Reconstruction Method (DRM) assesses how people spend their time and how they experience the various activities and settings of their lives, combining features of time-budget measurement and experience sampling. Participants systematically reconstruct their activities and experiences of the preceding day with procedures designed to reduce recall biases. The DRM's utility is shown by documenting close correspondences between the DRM reports of 909 employed women and established results from experience sampling. An analysis of the hedonic treadmill shows the DRM's potential for well-being research.

Problem

Robert is on a ski vacation with his wife, while John is at home. He can’t even dream of a ski vacation with the two children, to say nothing of the expense. In fact, John would be quite happy just to have a good night sleep.

Do you think that Robert is happier than John?

What’s Happiness?

- Both subjective well being and day reconstruction would suggest that Robert is happier
- And yet…
- What’s happiness for you?
- How should we measure happiness for social policies?