Reasoning in Face of Uncertainty

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Does Rationality Imply Bayesianism?

- Most theorists: yes
- Will try to challenge that
- We'll need to define the concepts
    - With minor digressions
- Implications for economic theory
- And some musings on how and why

Rationality

- Older concept: "Rational Man" should do...
- In neoclassical economics: only consistency
- An even more subjective view: which consistency?
- Rationality as robustness
- Weaknesses (?): subjective, empirical, not monotonic in intelligence
- Defense

Digression I: Objectivity and Subjectivity

- Anscombe-Aumann
- Schmeidler's example
- Objectivity as second-order subjectivity
- Habermas's notion of "communicative rationality"

Objective and Subjective Rationality

- A decision maker is defined by two relations $\left(\succsim^*, \succsim^{\hat{}}\right)$
- $\succsim^*$ – can convince "any reasonable decision maker" that it is right
- $\succsim^{\hat{}}$ – cannot be convinced that it is wrong
- Clearly, $\succsim^* \subset \succsim^{\hat{}}$

Objective and Subjective Rationality – An Example

- Personally, I think that both $\succsim^*$ and $\succsim^{\hat{}}$ should be transitive, but the test is different
  - $\succsim^{\hat{}}$ – "wouldn't it be embarrassing to be caught $f \succ \hat{} g \succ \hat{} h \succ \hat{} f$"?
  - $\succsim^*$ – "If you have a proof that $f \succsim^* g$
  - and a proof that $g \succsim^* h$
  - – there's a proof that $f \succsim^* h$ right there!"
- Objective rationality is a property of an instance $f \succsim^* g$
- Subjective rationality – of the entire relation $\succsim^{\hat{}}$

The Bayesian Approach

- Formulate state space
- All uncertainty resolved by the state
- Formulate a prior probability
- Update by Bayes's rule

Digression II: Classical Statistics

- Say, hypotheses tests:
  - H_0 : the defendant is innocent
  - H_1 : the defendant is guilty
- No probability on either hypothesis
  - "significance", "confidence" – derived from but are not probabilities
- The Bayesian alternative

Reconciling Classical and Bayesian Statistics

- Classical: attempts to be objective, no intuition
- Bayesian: attempts to incorporate intuition and hunches
- Classical – for making a point (to others)
- Bayesian – for making a decision (for oneself)

Pascal's Wager (1670) – I

- Blaise Pascal (1623-1662) analyzed the choice of becoming a believer
  - Based mostly on Ian Hacking (1975), "The Emergence of Probability"

	God is	God is not
Become a believer		
Forget about it		

- Pascal made it clear how one can become a believer
- First contribution: the decision matrix

Pascal's Wager – II

- First argument:

	God is	God is not
Become a believer	∞	0
Forget about it		0

- What have you got to lose?
- Second contribution: a dominant strategy

Pascal's Wager – III

- Second argument: well, in case you say

	God is	God is not
Become a believer	∞	0
Forget about it		$c > 0$

- The payoff awaiting you on Earth is finite $c < \infty$
- Hence you should "bet at all odds"
- Third contribution: expected utility maximization

Pascal's Wager – IV

- Wait a minute, what are these probabilities?

	God is	God is not
Become a believer	∞	0
Forget about it		$c > 0$

- These aren't empirical frequencies
- Fourth contribution: subjective probabilities
- Using the machinery developed for chance game to make sense of our intuition

Pascal's Wager – V

- And should you say that you don't know the probability...

	God is	God is not
Become a believer	∞	0
Forget about it		$c > 0$

- The argument works for any $\varepsilon \in (0, \bar{\varepsilon})$
- Fifth contribution: multiple probabilities

Pascal's Wager – Beyond Decision Theory

- Pascal did not say...

	God is	God is not
Become a believer	∞	0
Forget about it	$-\infty$	$c > 0$

- Invented positive marketing
- Got a lot of credit for the humanistic approach
  - See James Connor (2006) "Pascal's Wager: The Man Who Played Dice with God"

Bayes (1764)

- Thomas Bayes (1702-1761) wanted to prove that God is much more likely to exist than not
  - See Sharon McGrayne's (2011) "The theory that would not die"
- The argument was simple: we observe the world W and wonder about the existence of God G

$$P(W|G) = 1 \qquad P(W|\bar{G}) = \varepsilon$$

- But Bayes knew that to go from $P(W|G) = 1$ to $P(G|W)$ we need a prior $P(G) = p$

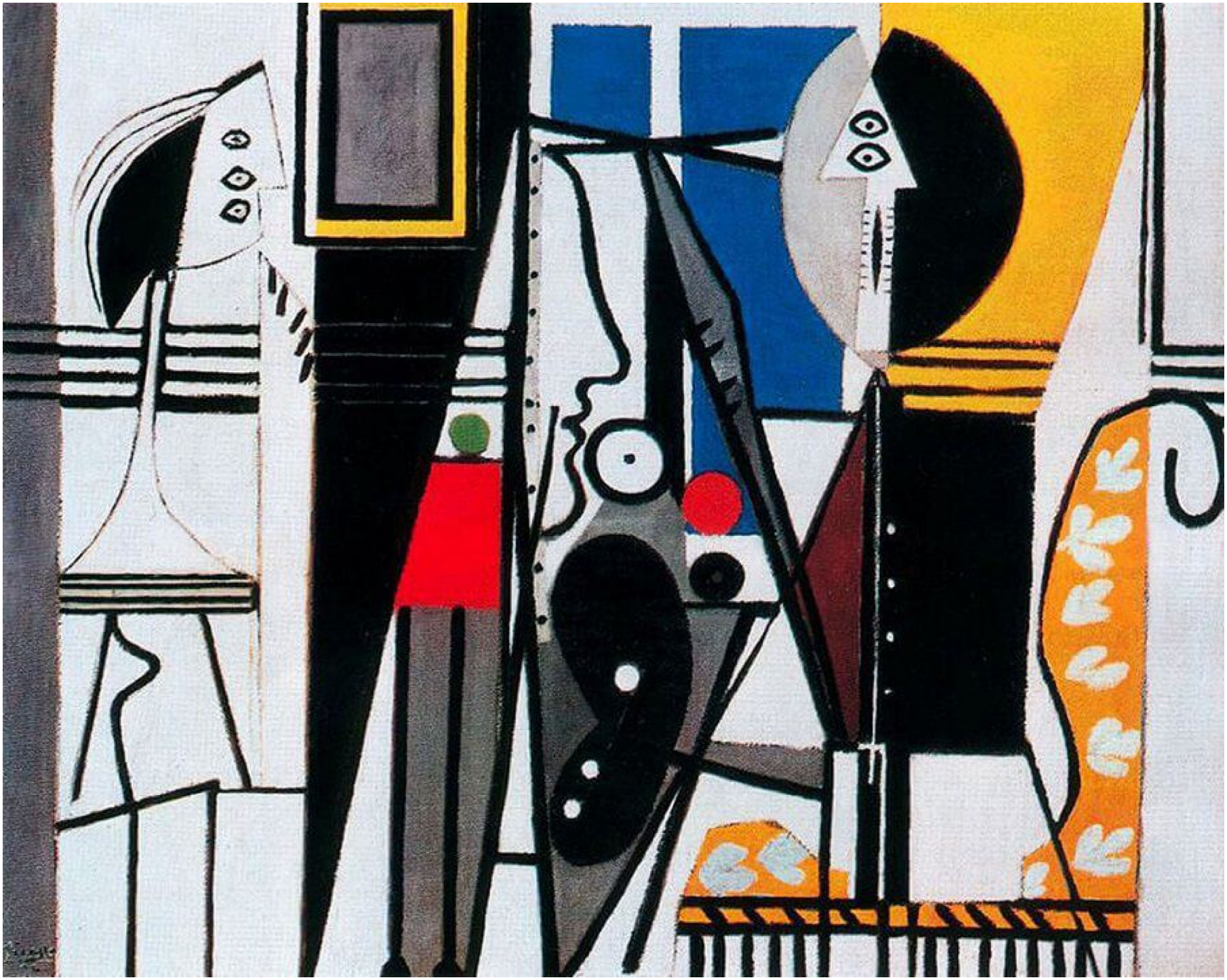
$$P(G|W) = \frac{pP(W|G)}{pP(W|G) + (1-p)P(W|\bar{G})} = \frac{p}{p + (1-p)\varepsilon}$$

Rationality and Bayesianism

- Pascal (1670)
- Bayes (1764)
- Ramsey (1926) and de Finetti (1931, 1937)
  - vs. Knight (1921) and Keynes (1921)
- von Neumann-Morgenstern (1944/7)
- Savage (1954)
- Anscombe-Aumann (1963)

Axiomatizations

- Conditions on presumably observed data that imply certain models
- Observability – along the lines of logical positivism (see Moscati)
- For example:
  - $\succsim$ complete and transitive $\Leftrightarrow$ can be represented by max u
  - (up to details)
- These are rhetorical results
  - Like existence, impossibility
  - Part of the discourse of theorists



The Bible (Savage, 1954)

- $F = X^S = \{f \mid f : S \to X\}$
- **P1** $\succsim$ is a weak order
- **P2** $f^h_{A^c} \succsim g^h_{A^c}$ iff $f^{h'}_{A^c} \succsim g^{h'}_{A^c}$
- **P3** $x \succsim y$ iff $f^x_A \succsim f^y_A$
- **P4** $y^x_A \succsim y^x_B$ iff $w^z_A \succsim w^z_B$
- **P5** $\exists\ f \succ g$
- **P6** $f \succ g$ $\exists$ a partition of S, $\{A_1, ..., A_n\}$ $f^h_{A_i} \succ g$ and $f \succ g^h_{A_i}$

Savage's Theorem

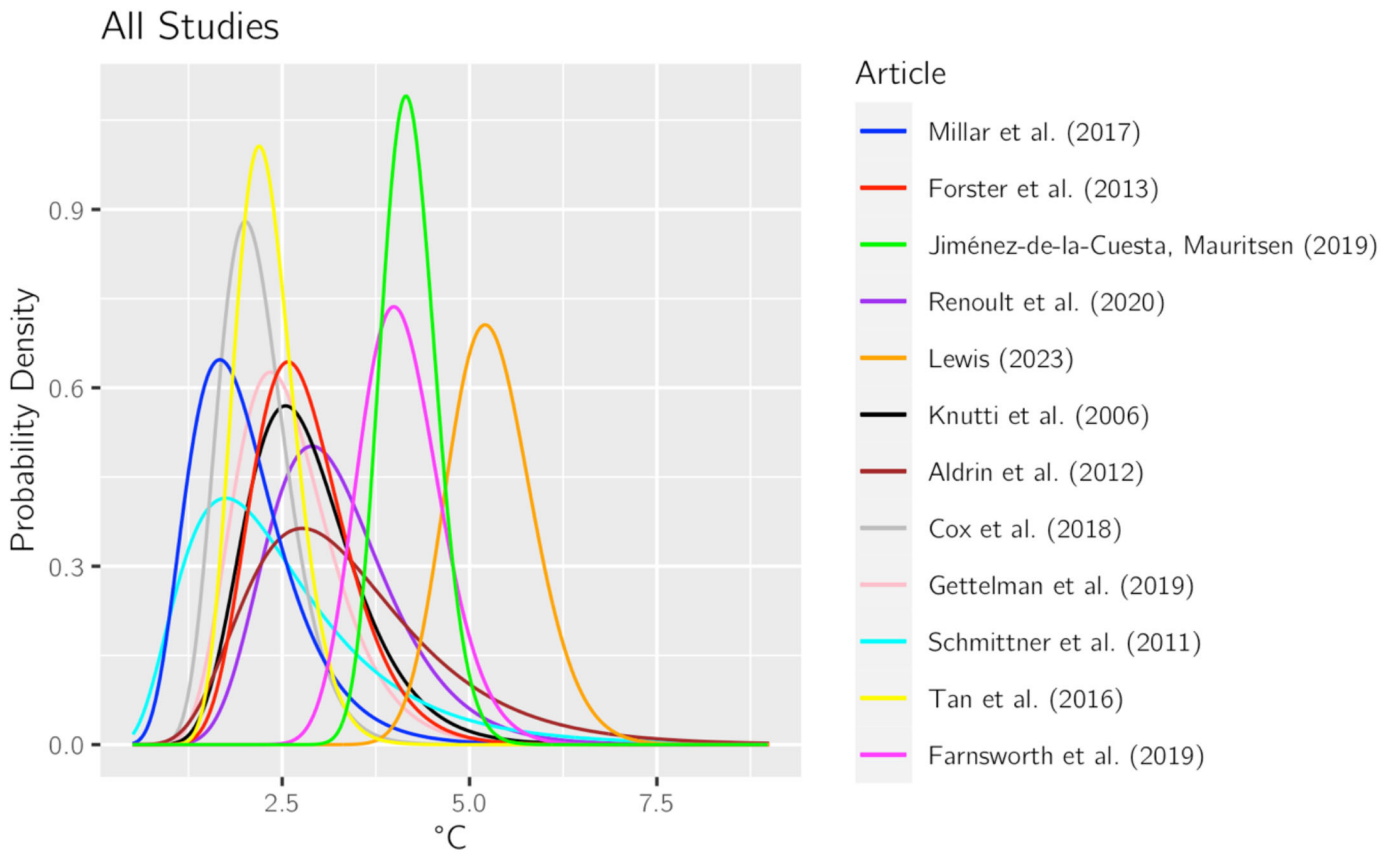
- Assume that X is finite. Then $\succsim$ satisfies P1-P6 if and only if there exist a non-atomic finitely additive probability measure μ on S $(=(S, 2^S))$ and a non-constant function $u : X \to \mathbb{R}$ such that, for every $f, g \in F$

$$f \succsim g \quad \text{iff} \quad \int_S u(f(s))d\mu(s) \geq \int_S u(g(s))d\mu(s)$$

Furthermore, in this case μ is unique, and u is unique up to positive linear transformations.

And yet...

- If it's so rational, why isn't it objective?
- Are all Arbodytes Cyclophines?
- What is the probability of
  - A coin coming up Head?
  - A car being stolen?
  - A surgery succeeding?
  - A war erupting?
- The Bayesian approach is good at representing knowledge, poor at representing ignorance



Data from a meta-analysis of Sven Kuron

(HEC, Paris, MA thesis, 2023)

How Can We Reconcile These?

- A key is the interpretation of "a state"
- Pretty modest in de Finetti
- Mixed in Savage
- Harsanyi, Aumann
- Newcombe: also causal relationships
- Monty Hall: also the way information is imparted
- A problem for a behavioral derivation

Objective Probabilities

- Exist in simple cases (iid)
- Can be defined with identicality, as long as causal independence is retained
- Rule-based approaches: logit
- Case-based approaches: empirical similarity
- But none extends to the cases of wars, stock market crashes...

Alternatives to the Bayesian Approach

- Schmeidler (1989): non-additive probabilities (capacities)
- Integration by Choquet's integral
- Maxmin EU: there exists a set of probabilities C such that

$$V(f) = \min_{P \in C} \int_S u(f(s))\, dP(s)$$

Other Multiple-Priors Models

- Nau, Klibanoff-Marinacci-Mukerji: "smooth preferences"

$$\varphi : \mathbb{R} \to \mathbb{R}$$

$$\int_{\Delta(S)} \varphi \left(\int u(f)\, dp \right) d\mu$$

- Maccheroni-Marinacci-Rustichini: "variational preferences"

$$V(f) = \min_{P \in \Delta(S)} \left\{ \int_S u(f(s))\, dP(s) + c(P) \right\}$$

Incomplete Preferences

- Bewley:

$$\begin{array}{rcl} f & \succ & g \\ & & iff \\ \forall p & \in & C \\ \int_S u\left(f\left(s\right)\right) dP\left(s\right) & > & \int_S u\left(g\left(s\right)\right) dP\left(s\right) \end{array}$$

- Fits the "objective rationality" notion
- Can be combined with the maxmin criterion as "subjective rationality"

Where Do Probabilities Come From?

Case-Based Beliefs with David Schmeidler

- Case-based decision theory
- Case-based probabilities
- Case-based selection of theories

Where Do Sets of Probabilities Come From?

Joint with Stefania Minardi and Fan Wang

- This is basically a statistical problem
- Let's look at observations and see which theories make more sense
- If we study the belief formation process, we may have more agreement